3. Evolutionary Nash program

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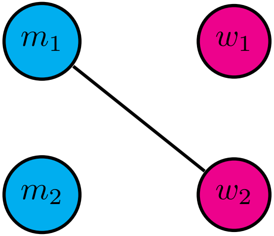
- The Evolutionary Nash Program works to link evolutionary game theory and cooperative game theory.
- Dynamic models of cooperative games.
- Understanding cooperative solution concepts in terms of the processes that can lead to them.

Matching

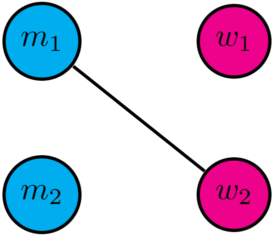
Stable matches, dynamics, one-shot principle and evolutionary axiomatization.

Marriage problem



- Set of men $M = \{m_1, \ldots, m_k\}$
- Set of women $W = \{w_1, \ldots, w_l\}$
- Players $N = M \cup W$
- Matchings G, undirected bipartite networks
- Each player matched to ≤ 1 other player.
- $g(i)$ is the partner of i at $g \in G$.
- $g(i) = \emptyset$ indicates that i is single at $g \in G$.

Marriage problem



- Player i has utility $u_i(g), g \in G$.
- Players only get utility from own partner.
  - — If $g(i) = g'(i)$, then $u_i(g) = u_i(g')$
- Strict preferences over partners
  - — If $g(i) \neq g'(i)$, then $u_i(g) \neq u_i(g')$

	w_1	w_2
m_1	$8, 2$	$7, 9$
m_2	$5, 6$	$7, 5$

Payoff of zero when unmatched.

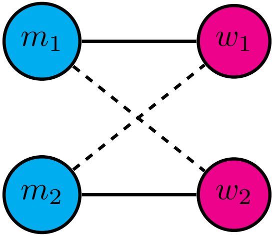
Stable matchings

Definition (Stable matchings)

A matching g is stable if

1. There are no i, $j = g(i)$ such that i prefers to be single than matched to j.
2. There are no i, j who prefer one another to their partners at g.

Let $S \subseteq G$ be the set of stable matchings.



	w_1	w_2
m_1	$8, 2$	$7, 9$
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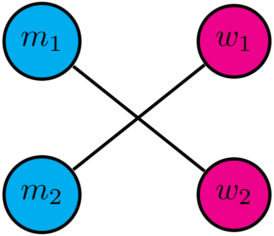
Payoff of zero when unmatched.

$S = \{g_M, g_W\}$

g_M ━━━

g_W - - - -

Rawlsian stable matchings



Definition (Rawlsian stable matchings)

The set of Rawlsian stable matchings is

$$Ra = \arg\max_{g \in S} \min_{i \in N} u_i(g)$$

Rawlsian stable matchings are the stable matchings that maximize the lowest payoff amongst all players.

	w_1	w_2
m_1	$8, 2$	$7, 9$
m_2	$5, 6$	$7, 5$

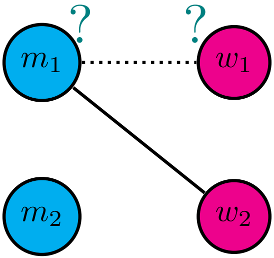
Payoff of zero when unmatched.

$Ra = \{g_W\} \subset \{g_M, g_W\} = S$

Matching dynamics

Consider the following dynamic, $t = 1, 2, \ldots$.

- State space is G.
- Every period, a man and a woman meet.
- If currently matched to one another, they consider separating.
  - Separate if at least one accepts separation.
- Otherwise, they consider leaving existing partners and matching with one another.
  - Match if both accept this.



	w_1	w_2
m_1	$8, 2$	$7, 9$
m_2	$5, 6$	$7, 5$

Payoff of zero when unmatched.

Matching dynamics

From state g^t, faced with the prospect of g', a player i will

- Accept g' with high probability if $u_i(g') > u_i(g^t)$.
- Accept g' with probability $\varepsilon^{\varphi(u_i(g^t),u_i(g'))}$ if $u_i(g') < u_i(g^t)$.

Definition (Condition dependence)

Behavior is condition dependent if φ is such that, for all $u, u', v, v' \in \mathbb{R}$, $u > u'$, $v > v'$, $u > v$, we have that $\varphi(u, u') > \varphi(v, v')$.

Acceptance of a detrimental change is less likely when current payoffs are higher.

Condition dependence and Rawlsian matchings

For sets M, W, let $\mathcal{U}$ be the set of all possible utilities.

Let SS denote the set of stochastically stable matchings.

Theorem

1. *If behavior is condition dependent, then* $\forall\, u \in \mathcal{U}$, *we have* $SS \subseteq Ra$.
2. *If behavior is not condition dependent, then* $\exists\, u \in \mathcal{U}$ *such that* $SS \not\subseteq Ra$.

That is, an axiomatization of Rawlsian stable matchings in terms of behavioral rules.

Condition dependence and Rawlsian matchings

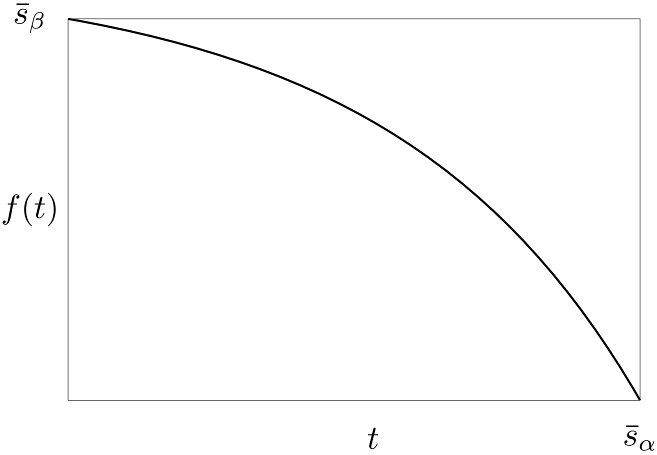
- To leave a stable matching requires some player to accept a change that leads to a lower payoff.
- Under condition dependence, it is easier to accept such a change when current payoffs are low.
- The makes Rawlsian stable matchings the stable matchings that are hardest to leave with an initial mistake (one-shot stability).
- There exists a result that, in this type of matching problem, stochastically stable matchings are contained within the one-shot stable matchings.

Bargaining solutions

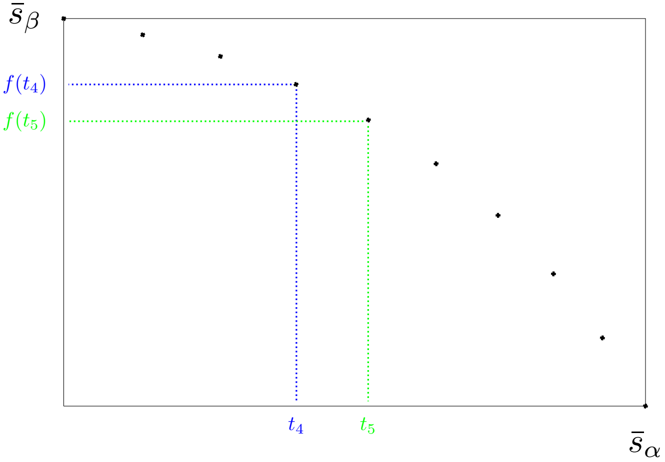
A characterization of solutions in terms of dynamic processes.

Bargaining frontier

- Players α and β
- Bargaining frontier $f(\cdot)$
- Pareto allocations
    - — α gets $t \in [0, \bar{s}_\alpha]$
    - — β gets $f(t) \in [0, \bar{s}_\beta]$



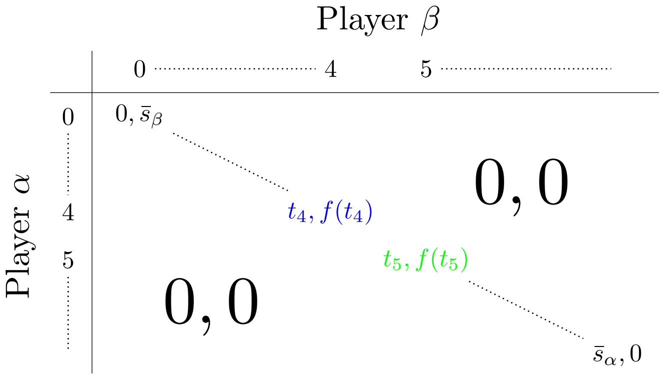
Bargaining frontier



- Discretize frontier
- Take payoff pairs

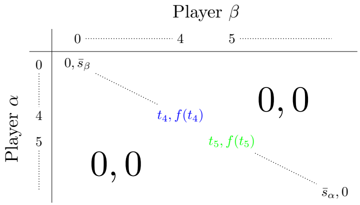
Coordination game

- Put payoffs on diagonal of coordination game
- Zero payoff off-diagonal



Population dynamics

- Consider two populations, α and β
- Each population has size N
- State is strategies for every player
- Periods $t = 1, 2, \ldots$

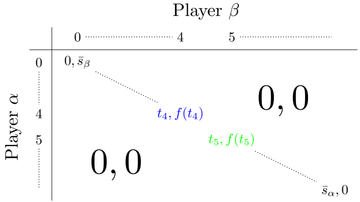


Population dynamics

- A player updating at time t plays a perturbed best response to the mixture given by the shares of strategies in the other population at time $t-1$.
- Consider four types of perturbations, varying on two dimensions
  - Uniform vs. Logit (have already seen these)
  - Intentional vs. Unintentional

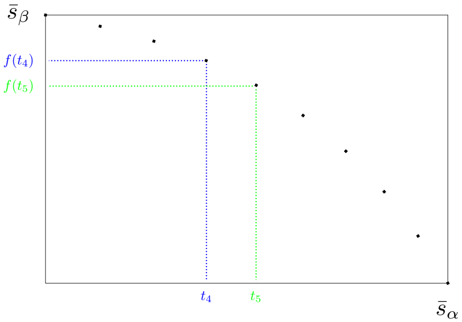
Population dynamics

- Unintentional means no change (!)
- Intentional truncates perturbations so that a player never asks for less than his best response.
- For example, if an α-player's best response is strategy 4, then under intentional perturbations
  - — may play strategy 5 (as a perturbation)
  - — will never play strategy 3



Population dynamics

- Unintentional favours big transitions, e.g.
  - α-players demand nothing
  - β-players respond demanding everything
  - $\bar{s}_\alpha$ and $\bar{s}_\beta$ matter
- Intentional favours small transitions, e.g.
  - α-players demand a little more
  - β-players respond demanding a little less
  - slope of $f(\cdot)$ matters
- Logit favours perturbations by those currently receiving low payoffs

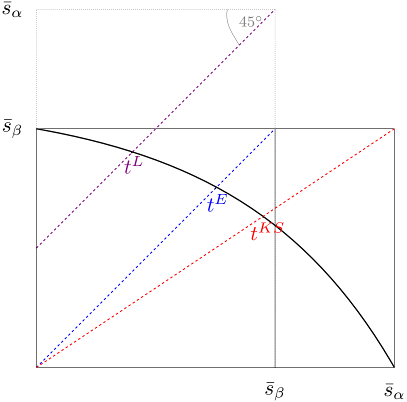


Convergence to bargaining solutions

Theorem

For fine discretization, large N, SS states approximate the following bargaining solutions

	Unintentional	**Intentional**
Uniform	*Kalai-Smorodinsky*	*Nash*
Logit	*Logit b.s.*	*Egalitarian*



Before we go...

- Of course, there is more to the Evolutionary Nash Program.
- General cooperative games, recontracting, convergence to the core, selection within the core, general behavioral rules in matching, matching with transferable utility.
- In general, the question of how aspects of culture arise and persist, embodied in collective institutions and conventions.
- Evolution of the constraints themselves: individual constraints, collective constraints, the traits that shape behavior.

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For references, see reading list.