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Statistical inference in evolutionary dynamics. (English. English summary)

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Each player from a unit mass population of players has two actions, A and B. Each player's payoff depends on the share of the population who plays A and an idiosyncratic component (i.e., player types). Specifically, the payoff from action B is set to zero, and the payoff from action A is given by $u(\theta,\alpha)=\theta-f(\alpha)$, where θ is the agent's idiosyncratic preference and $f(\alpha)$ is the cost of taking action A when the proportion of players playing A is α. $f(\cdot)$ is assumed to be continuous, so that for α in small enough $N\subseteq[0,1]$, a player is always playing a coordination game or anti-coordination game with every member of the population (uniform interaction/playing the field).

Each player's action choice follows the following individualistic dynamic. A player samples k other players from the population. From the share of these players that play A, he then generates a distribution over the share of players in the population as a whole who play A. This distribution is assumed to be monotonic in the number of sampled players who play A. This is referred to as *statistical inference*. He then plays a best response to this distribution.

Comparison is carried out between the *aggregate dynamic*, for which the state variable is the share of players who are choosing A, and a *Bayesian dynamic*, for which the state variable is the share of the players of each type who are choosing A. Theorems 1 & 2 show that these approaches effectively lead to the same outcomes.

Theorem 1 shows that the Bayesian dynamic converges if and only if the aggregate dynamic converges. The proof is an exercise in showing that convergence in the aggregate number of players choosing A leads to convergence in (probabilistic) best responses for each type of player. Theorem 2 uses monotonicity implied by the coordination/anti-coordination aspect of the problem to show convergence.

Subsequent results then study the dynamics for particular games, such as anti-coordination games (Proposition 1) and coordination games (Theorem 4).

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[References]

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