MR4503417 91A22 91A11

Izquierdo, Segismundo S. (E-VALD-INO); **Izquierdo, Luis R.** (E-BUR-MGE)

Stability of strict equilibria in best experienced payoff dynamics: simple formulas and applications. (English. English summary)

J. Econom. Theory **206** (2022), *Paper No.* 105553, 29 *pp.*

Consider a population comprising a unit mass of agents. These agents are matched in groups of size $p \in \mathbb{N}$ to play a p-player symmetric normal form game with strategy set $S = \{1, \ldots, n\}$ and payoff function $U: S^p \to \mathbb{R}$. The *population state* x is a vector with n elements. Element x_i denotes the fraction of agents using strategy $i \in S$.

When an agent updates his strategy he will test his current strategy against alternative strategies. Each strategy that is tested will be played in $\kappa \in \mathbb{N}$ independent *trials* against random draws of $p-1$ other players from the population. The strategy that obtains the greatest total payoff across all trials is then selected by the agent. If there is a tie, then some tie-breaking rule is applied.

Let each agent in the population update at Poisson rate 1. Let $w_i(x)$ denote the probability that an agent chosen uniformly at random from the population will select strategy i when he follows the above procedure. This gives the *best experienced payoff dynamics* $\dot{x}_i = w_i(x) - x_i$, the expected motion of the population from each state.

Let $s \in S$ be a strategy such that $(s, \ldots, s)$ is a strict symmetric Nash equilibrium of the game. Let e_s be the state at which every agent in the population plays s. Let v^s_{ij} be the total payoff to $i \in S$ when it is matched exclusively to players playing s in $\kappa - 1$ trials, and in the remaining trial matches with $(p-1)$ players playing s and a single player playing $j \in S$. These are the important payoffs to consider when almost everyone in the population plays strategy s, as the probability of matching with two or more players who do not play s is an order of magnitude lower than the probability of matching with one player who does not play s.

Let S_2 be the set of strategies that obtain the second-best payoff v^s_{ts} when playing against s players. That is, $S_2 = \arg\max_{i \neq s} v^s_{is}$.

Strategy $j \in S$ is *s-stabilizing* in $J \subseteq S \setminus \{s\}$ if

1. $v^s_{ij} < v^s_{ss}$ for all $i \in J$, and
2. if $S_2 \cap J \neq \varnothing$, then $v^s_{sj} > v^s_{ts}$.

Intuitively, in a neighborhood of e_s, if j is s-stabilizing in J, then j does not help any other strategy in J (including itself) to destabilize e_s. If a revising agent samples one player using strategy $j \neq s$, then condition (1) ensures that if the j-player is met when testing strategy i, then i does worse than s, so that s is selected over i. Similarly, condition (2) ensures that if the j-player is met when testing strategy s, then no strategy in J is selected.

The above suggests a procedure that gives a sufficient condition for stability of e_s. If a strategy i is s-stabilizing in $S \setminus \{s\}$, then it does not help any other strategy to destabilize e_s. We remove i from the strategy set and consider the reduced problem with strategy set $S \setminus \{i\}$. If, continuing in this manner, it is possible to remove all strategies in $S \setminus \{s\}$, then it must be that s is stable under the best experienced payoff dynamics. This is Proposition 4.2, one of the main results of the paper.

Another main contribution, Proposition 4.1, establishes a result in the other direction, using a weaker version of s-stabilizing strategies, *potentially s-stabilizing* strategies. If some strategy survives the iterated deletion of such strategies, it is shown that e_s is unstable. *Jonathan Newton*

[References]

1. Arigapudi, S., Heller, Y., Milchtaich, I., 2021. Instability of defection in the prisoner's

dilemma under best experienced payoff dynamics. J. Econ. Theory 197, 105174. MR4305938

2. Arigapudi, S., Heller, Y., Schreiber, A., 2022. Sampling dynamics and stable mixing in hawk-dove games. Working paper available at https://arxiv.org/abs/2107.08423.
3. Benaïm, M., Weibull, J.W., 2003. Deterministic approximation of stochastic evolution in games. Econometrica 71, 873–903. MR1983230
4. Berkemer, R., 2008. Disputable advantage of experience in the traveler's dilemma. Unpublished manuscript, Technical University of Denmark. Abstract in International Conference on Economic Science with Heterogeneous Interacting Agents, Warsaw, 2008.
5. Brown, G.W., von Neumann, J., 1950. Solutions of games by differential equations. In: Kuhn, H.W., Tucker, A.W. (Eds.), Contributions to the Theory of Games I. In: Annals of Mathematics Studies, vol. 24. Princeton University Press, Princeton, pp. 73–79. MR0039220
6. Cárdenas, J.C., Mantilla, C., Sethi, R., 2015. Stable sampling equilibrium in common pool resource games. Games 6, 299–317. MR3407806
7. Chmura, T., Güth, W., 2011. The minority of three-game: an experimental and theoretical analysis. Games 2, 333–354. MR2845106
8. Crawford, V.P., 1991. An "evolutionary" interpretation of Van Huyck, Battalio, and Beil's experimental results on coordination. Games Econ. Behav. 3 (1), 25–59.
9. Heller, Y., Mohlin, E., 2018. Social learning and the shadow of the past. J. Econ. Theory 177, 426–460. MR3849760
10. Izquierdo, L.R., Izquierdo, S.S., 2022. BEP-TCG. Software available at https://doi.org/10.5281/zenodo.6975573.
11. Izquierdo, L.R., Izquierdo, S.S., Rodriguez, J., 2022. Fast and scalable global convergence in single-optimum decentralized coordination problems. IEEE Trans. Control Netw. Syst. https://doi.org/10.1109/TCNS.2022.3181545.
12. Izquierdo, L.R., Izquierdo, S.S., Sandholm, W.H., 2018. EvoDyn-3s: a Mathematica computable document to analyze evolutionary dynamics in 3-strategy games. SoftwareX 7, 226–233.
13. Izquierdo, S.S., Izquierdo, L.R., 2021. "Test two, choose the better" leads to high cooperation in the centipede game. J. Dyn. Games. MR4505964
14. Kosfeld, M., Droste, E., Voorneveld, M., 2002. A myopic adjustment process leading to best reply matching. J. Econ. Theory 40, 270–298. MR1920973
15. Kreindler, G.E., Young, H.P., 2013. Fast convergence in evolutionary equilibrium selection. Games Econ. Behav. 80, 39–67. MR3061229
16. Mantilla, C., Sethi, R., Cárdenas, J.C., 2020. Efficiency and stability of sampling equilibrium in public goods games. J. Public Econ. Theory 22 (2), 355–370.
17. Miękisz, J., Ramsza, M., 2013. Sampling dynamics of a symmetric ultimatum game. Dyn. Games Appl. 3, 374–386. MR3100615
18. Osborne, M.J., Rubinstein, A., 1998. Games with procedurally rational players. Am. Econ. Rev. 88, 834–847.
19. Osborne, M.J., Rubinstein, A., 2003. Sampling equilibrium, with an application to strategic voting. Games Econ. Behav. 45 (2), 434–441. Special Issue in Honor of Robert W. Rosenthal. MR2023673
20. Oyama, D., Sandholm, W.H., Tercieux, O., 2015. Sampling best response dynamics and deterministic equilibrium selection. Theor. Econ. 10, 243–281. MR3316456
21. Perko, L., 2001. Differential Equations and Dynamical Systems, third edition. Springer, New York. MR1801796
22. Rowthorn, R., Sethi, R., 2008. Procedural rationality and equilibrium trust. Econ. J. 118, 889–905.

23. Salant, Y., Cherry, J., 2020. Statistical inference in games. Econometrica 88 (4), 1725–1752. MR4131235
24. Sandholm, W.H., 2001. Almost global convergence to p-dominant equilibrium. Int. J. Game Theory 30, 107–116. MR1863717
25. Sandholm, W.H., 2010. Population Games and Evolutionary Dynamics. MIT Press, Cambridge. MR2560519
26. Sandholm, W.H., 2014. Local stability of strict equilibria under evolutionary game dynamics. J. Dyn. Games 1 (3), 485–495. MR3306265
27. Sandholm, W.H., Izquierdo, S.S., Izquierdo, L.R., 2019. Best experienced payoff dynamics and cooperation in the centipede game. Theor. Econ. 14 (4), 1347–1385. MR4040340
28. Sandholm, W.H., Izquierdo, S.S., Izquierdo, L.R., 2020. Stability for best experienced payoff dynamics. J. Econ. Theory 185, 104957. MR4028211
29. Sawa, R., Wu, J., 2021. Statistical inference in evolutionary dynamics. Available at https://doi.org/10.2139/ssrn.3767635.
30. Sethi, R., 2000. Stability of equilibria in games with procedurally rational players. Games Econ. Behav. 32, 85–104. MR1771445
31. Sethi, R., 2021. Stable sampling in repeated games. J. Econ. Theory 197, 105343. MR4309440
32. Smith, M.J., 1984. The stability of a dynamic model of traffic assignment—an application of a method of Lyapunov. Transp. Sci. 18, 245–252. MR0878966
33. Spiegler, R., 2006a. Competition over agents with boundedly rational expectations. Theor. Econ. 1 (2), 207–231.
34. Spiegler, R., 2006b. The market for quacks. Rev. Econ. Stud. 73 (4), 1113–1131. MR2260758
35. Taylor, P.D., Jonker, L., 1978. Evolutionarily stable strategies and game dynamics. Math. Biosci. 40, 145–156. MR0489983
36. Van Huyck, J.B., Battalio, R.C., Beil, R.O., 1990. Tacit coordination games, strategic uncertainty, and coordination failure. Am. Econ. Rev. 80 (1), 234–248.
37. Weibull, J.W., 1995. Evolutionary Game Theory. MIT Press, Cambridge. MR1347921

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.