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Cui, Zhiwei (PRC-PUC-SEC); **Jiang, Ge** (PRC-NAN-SEC); **Shi, Fei** (PRC-JTU-EMG)

Size-dependent minimum-effort games and constrained interactions. (English. English summary)

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There is a set of players $I = \{1, \ldots, n\}$ and a set of possible real-numbered efforts $E = \{e^1, \ldots, e^\rho\}$ with $0 < e^1 < e^2 < \cdots < e^\rho$.

At time $t \in \mathbb{N}$, each player $i \in I$ is associated with an effort level $e_i \in E$ and $N_i^{\rm out} \subset I \setminus \{i\}$, $|N_i^{\rm out}| \leq M$, the set of players to whom he chooses to *link*. It is assumed that $n > 2M + 1$. Let $N_i^{\rm in}$ be the set of players who choose links to i. Let $N_i = N_i^{\rm out} \cup N_i^{\rm in}$.

Given efforts and links, a player's payoff is given by summing three terms.

(1) $N_i + 1$ multiplied by $\min_{j \in N_i \cup \{i\}} e_j$, the lowest effort taken amongst all the players in $N_i \cup \{i\}$.
(2) Cost of effort, $-\delta\, e_i$. It is assumed that $0 < \delta < 1$.
(3) Cost of linking, $-\gamma\, |N_i^{\rm out}|$. It is assumed that γ is sufficiently small for subsequent arguments to hold.

Consider a dynamic process where each period a player is randomly chosen to update his effort and links. He updates to maximize his payoff, randomizing over ties.

If there are players making different effort choices, let player i make the lowest effort and player k make some other effort. When i updates, he either (i) increases his effort, or (ii) retains the same effort and with positive probability links to k. If the latter case is realized and k then updates, k will reduce his effort, as there is no gain when he exerts effort greater than $\min_{j \in N_k \cup \{k\}} e_j = e_i < e_j$. In this manner, we can reach a state at which all players choose the same effort. From such a state, players will continue to choose the same effort, although the links they choose may change over time due to indifference (Proposition 1).

A perturbed version of the dynamic is then considered in which an updating agent, with small probability ε, chooses a random effort instead of a best response. From a state at which all players choose the same effort e', let player i randomly choose a lower effort e. If i links to j, then it is possible for j to subsequently best respond, dropping his effort from e' to e. In this manner, it is possible for all players to reduce their efforts following only a single non-best response. In contrast, non-best responses to higher efforts induce no such cascade. Consequently, for small enough ε, the invariant distribution of the perturbed dynamic places most of its mass on states at which all players choose the lowest possible effort e^1 (Proposition 2). *Jonathan Newton*