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Ordinal status games on networks. (English. English summary)
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Consider a finite set of *players* N, $|N| \geq 2$. Each $i \in N$ has strategy set X_i, which is a closed interval in $\mathbb{R}$. The set of *strategy profiles* is $X_N = \prod_{i \in N} X_i$. Given a strategy profile, a status mapping $\sigma_i \colon X_N \to S$ gives the status $s \in S$ of player i from the finite ordered set S. *Utility* $U_i \colon X_i \times S \to \mathbb{R}$ is such that $U_i(x_i, s)$ is strictly increasing in S and single-peaked in x_i. That is, for all $i \in N$ and $s \in S$, there exists $\widehat{x}_i^s \in X_i$ such that $U_i(x_i, s)$ strictly increases in x_i when $x_i \leq \widehat{x}_i^s$ and strictly decreases when $x_i \geq \widehat{x}_i^s$.

A *network status game* has the status of player i increasing in the order rank of his strategy x_i with respect to the strategies of the other players. That is, status is increasing in the number of players j for whom $x_j \leq x_i$. A game is *dichotomic* if there are only two statuses, bottom $\perp$ and top $\top$, ordered so that $\top$ is greater than $\perp$. For such games, for a given player i and strategy profile x_N, it is possible to define $\xi_i(x_N)$ to be the minimum strategy that i could play that would obtain him the status $\top$, keeping the strategies of the other players fixed. This observation, together with the monotonicity and single-peakness assumptions, is used to characterize the best response correspondence for the game (Lemma 4.1).

A *very strong equilibrium* is a strategy profile from which no coalition of players can adjust their strategies in such a way that no member's payoff decreases and at least one member's payoff strictly increases. The main results of the paper under review are that every dichotomic network status game has a very strong equilibrium (Theorem 4.3) and that if a procedure is followed in which at each step one player adjusts his strategy to obtain a higher payoff, then the procedure terminates at a Nash equilibrium (Theorem 4.4).

The proof of Theorem 4.3 proceeds constructively. The starting point is the strategy profile at which all players $i \in N$ play $\widehat{x}_i^\top$, the optimal choice conditional on status being $\top$. Following this, players' strategies are adjusted in the directions in which they would like to adjust them, prioritizing those who would wish to decrease their strategies over those who would wish to increase them. This allows exploitation of the complementarity in optimal strategies to show that the process satisfies a form of monotonicity and terminates. Given the structure of the problem, it is then fairly straightforward to show that, from the obtained strategy profile, for any given player to obtain a higher payoff, another player must change his strategy at a loss to himself. Therefore, a very strong equilibrium has been obtained. Theorem 4.4 is then proven by showing that cycles do not occur in this game when players adjust their strategies consecutively to obtain strictly higher payoffs. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.