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Constrained stability in two-sided matching markets. (English summary)

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This paper considers worker-firm matching problems in which there are finite sets of *workers* and *firms*. Each worker can be matched to at most one firm and has a preference ordering over firms. A worker may also prefer to remain unmatched than to be matched to a given firm. A firm f can match with at most $q_f > 0$ workers and has a preference ordering over sets of workers. Preferences are responsive in that if a firm prefers one worker to another, this will hold regardless of the other workers employed at the firm.

A matching of workers to firms is *individually rational* if (i) no worker would rather be unmatched than matched to his current firm, and (ii) no firm would prefer to have a vacancy instead of being matched to one of its current workers. Given a matching, a *blocking pair* is a firm and a worker such that (i) the worker prefers the firm to his current match, and (ii) the firm prefers the worker to one of its current workers (or to a vacancy if the firm's quota is currently unfilled). A matching is *stable* if it is individually rational and there is no blocking pair.

The paper considers a concept of *constrained stability* in which, for each matching, some of the potential blocking pairs do not have to be satisfied. However, it is assumed that all blocking pairs involving unmatched workers must be satisfied. In summary, constrained stability is weaker than stability. The paper notes (Proposition 3) that, unlike stable matchings, constrained stable matchings are not necessarily Pareto-efficient.

A *mechanism* asks every worker to state his preferences over firms, following which the mechanism chooses a matching of workers to firms. A mechanism is *strategy-proof* if no worker can ever gain (i.e., match to a better firm) by lying about his preferences. A mechanism is constrained stable if it outputs constrained stable matchings.

The paper considers the worker proposing a deferred acceptance (DA) mechanism. DA outputs a stable matching (hence also constrained stable) that is optimal for workers amongst all stable matchings. However, with appropriately chosen removal of blocking pairs, a constrained stable matching that is better for workers can be created (Proposition 2).

It turns out that a mechanism is constrained stable and strategy-proof if and only if it is DA (Theorem 1). The "if" statement follows immediately from the fact that DA outputs stable matchings and is strategy-proof [D. Gale and L. S. Shapley, Amer. Math. Monthly **69** (1962), no. 1, 9–15; MR1531503; A. E. Roth, Math. Oper. Res. **7** (1982), no. 4, 617–628; MR0686535].

The "only if" part is proved as follows. Conjecture a constrained stable and strategy-proof mechanism ψ that gives a different output to DA for some preferences. By [A. Abdulkadiroğlu, P. A. Pathak and A. E. Roth, Amer. Econ. Rev. **99** (2009), no. 5, 1954-1978], strategy-proofness implies that there must be a worker w_1 who for some preferences P prefers the output $DA(P)$ to the output $\psi(P)$. Let f_1 be the firm to which w_1 is matched at $DA(P)$.

These preferences P are restricted to give new preferences P' such that f_1 is the only firm that w_1 prefers to remaining unmatched. At $DA(P')$, w_1 is still matched to f_1 [F. Kojima and M. Manea, Econometrica **78** (2010), no. 2, 633–653; MR2656642].

Hence, $\psi(P')$ must leave w_1 unmatched. It follows from constrained stability of ψ that, at $\psi(P')$, f_1 must employ some worker w_2 that f_1 does not employ at $DA(P')$. Note that constrained stability implies that f_1 must prefer w_2 to w_1. Therefore, to avoid the existence of a blocking pair (f_1, w_2) that would contradict stability, at $DA(P')$ it must be that w_2 is matched to some f_2 that he prefers to f_1.

Now restrict the preferences of w_2 to give new preferences P'' such that the only match that w_2 prefers to remaining unmatched is f_2. Iterate the argument above to show that w_2 must then be unmatched at $\psi(P'')$ and that there must be some other worker w_3 who is matched at $\psi(P'')$ whose preferences we can further restrict. Noting that at each stage we add a new worker, finiteness of the set of workers implies that we reach a contradiction and the theorem is proven. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.