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Strategy-proofness of worker-optimal matching with continuously transferable utility. (English summary)

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This paper considers worker-firm matching problems in which there are finite sets of *workers* and *firms*. Each worker can be employed by at most one firm. Each firm has a *valuation* over each possible combination of workers it could hire, and each worker has a valuation over each possible firm that he could work for. The valuation of remaining unmatched is normalized to zero. A *contract* specifies which workers are matched to which firms, which workers remain unmatched, and the salaries of the matched workers. The *utility* of a worker under a contract is given by the worker's valuation of the firm to which he is matched, plus his salary. The utility of a firm is given by the firm's valuation of the workers to which he is matched, minus their salaries.

A contract is in the *core* if there does not exist any set of workers and firms that can strictly increase their utilities under the contract by rematching amongst themselves and choosing new salaries. A *core-selecting mechanism* (CSM) is a function from the valuations to the core. A CSM is *optimal* for worker  $w$  if it always selects a core contract that maximizes the utility of  $w$ . A CSM is *strategy-proof* for worker  $w$  if  $w$  cannot obtain a higher utility by misreporting his valuation so that the CSM selects a different contract. Thus, strategy-proofness tests the robustness of core matchings, and can be contrasted with dynamic robustness concepts such as stability under random utility shocks [J. Newton and R. Sawa, *J. Econom. Theory* **157** (2015), 1–27; MR3335933; B. Klaus and J. Newton, *J. Math. Econom.* **62** (2016), 62–74; MR3435745; H. H. Nax and B. S. R. Pradelski, *Internat. J. Game Theory* **44** (2015), no. 4, 903–932; MR3422869].

The theorem of the paper under review states that any CSM that is optimal for worker  $w$  is strategy-proof for  $w$ . The proof is by contradiction as follows:

Assume that a CSM is optimal but not strategy-proof for worker  $w$ . There must exist some true valuations for  $w$  over each firm, say  $v_w(\cdot)$ , such that  $w$  has an incentive to misreport. Let the CSM select contract  $A$  if  $w$  truthfully reports  $v_w$ . As  $w$  has an incentive to misreport, there exists some  $v'_w$  such that, when  $w$  reports  $v'_w$ , the CSM selects a contract  $A'$  that gives  $w$  strictly higher utility than  $A$ . Given that utility from any core contract (including  $A$ ) must be at least zero, worker  $w$  must be matched to a firm at  $A'$ . Let this firm be denoted  $f_w$ .

It follows that there exists some constant  $\eta > 0$  that is greater than the utility of  $w$  at  $A$  but less than the utility of  $w$  at  $A'$ . Let  $A''$  be the contract that the CSM selects if  $w$  reports  $v_w - \eta$ . It is shown that  $w$  must be unmatched at  $A''$  (otherwise  $A$  cannot be a  $w$ -optimal choice for the CSM at the true valuations).

Now consider  $\hat{v}_w$  such that  $\hat{v}_w(f_w) = v_w(f_w) - \eta$  and  $\hat{v}_w(f) = -\infty$  for  $f \neq f_w$ .  $A''$  remains in the core for these valuations. Furthermore, and this is the crux of the proof, it follows from the choice of  $\eta$  that  $A'$  is also within the core for these valuations and that  $w$  continues to obtain strictly positive utility from  $A'$ . However, by R. Jagadeesan, S. D. Kominers and R. Rheingans-Yoo [“Lone wolves in competitive equilibria”, Working Paper No. 18-055, Harvard Bus. Sch. Entrep. Manag., 2017, doi:10.2139/ssrn.3095542], if  $w$  obtains strictly positive utility at a core contract, then  $w$  is matched at every core

contract. This contradicts  $w$  being unmatched at  $A''$  and completes the proof.

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*