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Maximality in the farsighted stable set. (English summary)
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This paper considers a finite set of *players* N and a characteristic function v that assigns to each nonempty *coalition* $S \subseteq N$ a bounded set of *payoff* vectors $V(S) \in \mathbb{R}^S_+$. A *state* x is a pair $(\pi(x), u(x))$ such that $\pi(x)$ is a partition of N and for all $S \in N$, $u_S(x) \in V(S)$. For any pair of states x, y, the *effectivity correspondence* $E(x, y)$ equals the set of coalitions that have the power to change the state from x to y. It is assumed that (E.1) if $T \in \pi(x)$, $S \in E(x, y)$ and $S \cap T = \varnothing$, then $T \in \pi(y)$ and $u_T(x) = u_T(y)$. That is, T is left untouched when S changes the state from x to y. It is further assumed that (E.2) from a state x, any given coalition S has the power to change the state by breaking itself apart and arriving at a state y at which each piece of S obtains payoffs in accordance with the characteristic function.

A state y *farsightedly dominates* x if there exists $x = y^0, y^1, \dots, y^m = y$ and $S^1, \dots, S^m$ such that $S^k \in E(y^{k-1}, y^k)$ for $k = 1, \dots, m$, and every player in S^K obtains a strictly higher payoff at y than he does at y^{k-1}. A set of states F is a *farsighted stable set* if no state in F farsightedly dominates another state in F, and any state not in F is farsightedly dominated by a state in F.

Lemma 1 states that if y farsightedly dominates x, then it is possible to construct sequences above so that $S^1, \dots, S^{m-1}$ are pairwise disjoint and $\bigcup_{k=1}^{m-1} S^k \subseteq S^m$. This lemma is false, but can be made to hold [see J. Newton, "Maximality in the farsighted stable set revisited", working paper, 2020, doi:10.2139/ssrn.3590816; "Corrigendum to 'Maximality in the farsighted stable set'", 2020] under a condition specifying that when a coalition of players T is broken up by the participation of some of its players in a coalitional move by S, then the new coalitions and payoffs for the remainder of the players $T \setminus S$ depend on neither the coalitions and payoffs of players outside of T before the breakup, nor the coalitions and payoffs of players outside of $T \setminus S$ after the breakup, nor the identities of players in S who are not members of T.

A *history* is a finite sequence of states. A *negotiation process* σ maps each history h to a state $y(h)$ and a coalition $S(h)$ that implements this change. That is, if $x(h)$ is that last state in history h, then $S(h) \in E(x(h), y(h))$. A new history is then created by adding the new state to the old history. A state x is absorbing under σ if whenever $x(h) = x$, we have $y(h) = x$. σ is absorbing if, starting from any history, an absorbing state is reached. Let $x^\sigma(h)$ denote the absorbing state reached from h. An absorbing process is *coalitionally acceptable* if every player in $S(h)$ obtains at least a high payoff at $x^\sigma(h)$ as he does at $x(h)$. An absorbing process σ is *absolutely maximal* if there does not exist h, T, y such that $T \in E(x(h), y)$ and every player in T obtains a strictly higher payoff at $x^\sigma(h, y, T)$ than they do at $x^\sigma(h)$.

The main theorem of the paper (Theorem 1) states that if we assume two properties on states and payoffs, then given a farsightedly stable set F, we can construct a coalitionally acceptable, absolutely maximal process that has F as its set of absorbing states.

Property A states that if there are $a, b \in F$ such that player j obtains a strictly higher payoff at b than at a, then there exists $z \in F$ such that j obtains a weakly lower payoff at z than at a and all other players obtain a weakly higher payoff at z than at b.

Property B states that if $a, b \in F$ and all players in T obtain strictly higher payoffs at

b than at a, then $T \notin \pi(b)$.

The remainder of the proof proceeds as follows: Assume a blocking chain from x to $a \in F$, and another blocking chain from y to $b \in F$, and some set of players $T \in E(x, y)$ that all receive strictly higher payoffs at b than they do at a. By Property B it must be that $y \neq b$. Then using Property A and Lemma 1, a coalitionally acceptable process is constructed so that following a move by T from x to y, the process eventually transits to a state z such that at least one player $j \in T$ is no better off at z than he is at a, and all other players are at least as well off at z as they are at b. In other words, if j participates in an attempt to make the process end up at b rather than a, he can be punished and end up at z. All other players are willing to participate in such a punishment as they weakly prefer z to b. *Jonathan Newton*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.