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Conventions under Heterogeneous Behavioural Rules

JONATHAN NEWTON

Institute of Economic Research, Kyoto University

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Strategies of players in a population are updated according to the behavioural rules of agents, where each agent is a player or a coalition of players. It is known that classic results on the stochastic stability of conventions are due to an asymmetry property of the strategy updating process. We show that asymmetry can be defined at the level of the behavioural rule and that asymmetric rules can be mixed and matched whilst retaining asymmetry of the aggregate process. Specifically, we show robustness of asymmetry to heterogeneity within an agent (Alice follows different rules at different times); heterogeneity between agents (Alice and Bob follow different rules); and heterogeneity in the timing of strategy updating. These results greatly expand and convexify the domain of behavioural rules for which results on the stochastic stability of conventions are known.

Key words: Evolution, Conventions, Heterogeneity, Representative agent.

JEL Codes: C73, D01, D90.

1. INTRODUCTION

Lewis (1969) argued that conventions, regularities in the behaviour of members of a population when faced with a coordination problem, might arise from processes in which individuals in a population follow simple, adaptive behavioural rules. Young (1993a) and Kandori *et al.* (1993) formulated these ideas mathematically using the theory of Markov chains and showed, using the ideas of Freidlin and Wentzell (1984), that conventions can be ranked by their stability properties under given models of behaviour. Since then, the stability of conventions under a variety of behavioural rules has been considered (see Sandholm, 2010; Newton, 2018).

Methodologically, agents (individuals or coalitions) in a population update their strategies according to *behavioural rules*. This updating gives a Markov chain on the set of strategy profiles, the transition probabilities of which can be summarized by a *cost function*. Typically, the cost function then provides the input to a graph theoretic problem, the solution to which tells us the stability of our conventions, the most stable conventions being known as *stochastically stable* (Foster and Young, 1990). Peski (2010) showed that, if the cost function satisfies an asymmetry condition with respect to one of the conventions, then that convention is stochastically

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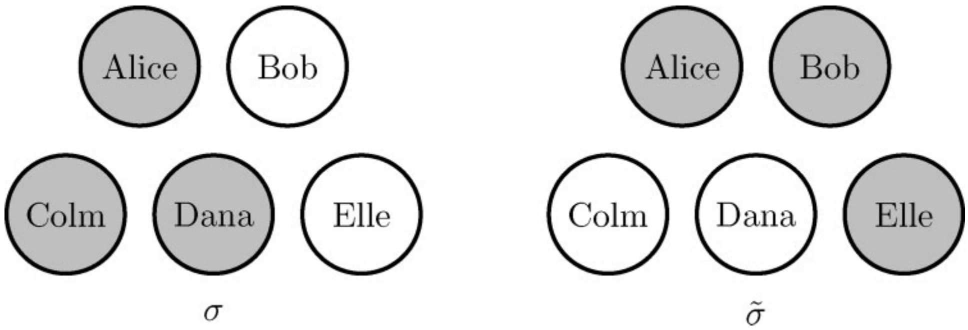

FIGURE 1

Asymmetry. Vertices shaded grey play A. Unshaded vertices play B. The set of players that play A at $\tilde{\sigma}$ (*i.e.* {*Alice*, *Bob*, *Elle*}) contains the set of players that play B at σ (*i.e.* {*Bob*, *Elle*}). Asymmetry towards A implies that further adoption of A starting from $\tilde{\sigma}$ is at least as likely as further adoption of B starting from σ. Formal mathematical definitions are given in Section 2.

stable.[1] Considering an environment with two strategies, A and B, *asymmetry* towards A roughly corresponds to the requirement that if strategy profiles $\sigma, \tilde{\sigma}$ are such that the set of players who play B at σ is a subset of the set of players who play A at $\tilde{\sigma}$, then switches to strategy B from σ are weakly less likely than switches to strategy A from $\tilde{\sigma}$. Thinking of A and B as alternative technologies, we can interpret this to mean that if technology A is at least as widespread at $\tilde{\sigma}$ as technology B is at σ, then further adoption of technology A from a starting point of $\tilde{\sigma}$ is at least as likely as further adoption of technology B from a starting point of σ (Figure 1).

Here, instead of considering asymmetry of the aggregate process, we disaggregate our analysis and consider asymmetry in behavioural rules. Specifically, for a given agent following a given behavioural rule, we can consider the asymmetry of the fictional process in which that agent updates and the remainder of the population never updates. This disaggregation allows us to consider three dimensions of heterogeneity. Firstly, we consider heterogeneity within an agent. It turns out that the set of asymmetric behavioural rules is convex. If two behavioural rules are asymmetric towards a given strategy, then a compound rule that sometimes follows one of the rules and sometimes follows the other is also asymmetric towards that strategy (Theorem 1). Secondly, we consider heterogeneity between agents. If every agent follows a behavioural rule that is asymmetric towards a given strategy, then the aggregate process is also asymmetric towards that strategy (Theorem 2). Finally, we consider heterogeneity in the timing of strategy updating. Asymmetry of the aggregate process does not depend on whether agents update their strategies at the same time or at different times (Theorem 3).

Consequently, when every agent follows an identical behavioural rule, we can obtain results on stochastic stability by showing asymmetry for a single representative agent. Many results from the literature can be recovered in this manner and results for many alternative behavioural rules can be derived.[2] Even better, we can mix and match agents who follow different behavioural rules, and if the agent-specific conditions for asymmetry are satisfied in each case, we are done. In

1. This result finally provided an affirmative answer to the long unanswered question of whether the strategy profile at which every player plays a risk dominant strategy is stochastically stable under the best response with uniform deviations behavioural rule for any network of interactions.

2. In particular, we consider behavioural rules, recover results, and extend results from Blume (1993), Blume (1996), Young (1993a), Kandori *et al.* (1993), Peski (2010), Ellison (2000), Ellison (1993), Dokumaci and Sandholm (2011), Norman (2009b), Maruta (2002), Blume (2003), Norman (2009a), Young (2011), Newton and Angus (2015),

Behavioral rule	**Properties relevant to asymmetry**	**Sections**
Payoff-difference based, self regarding	risk dominance	4
... + other regarding	... + altruistic risk dominance	4, 6.3
... + coalitions	... + payoff dominance	6.1
... + Kantian	... + payoff dominance	6.2
Imitative	payoff dominance + maximin	5

FIGURE 2

Properties relevant to asymmetry under classes of behavioural rule. Risk dominance and altruistic risk dominance are conditions on relative incentives. Payoff dominance and maximin are ordinal conditions. Payoff dominance and maximin together imply the "Lewis Conditions" on positive externalities of coordination. All behavioural rules and properties are formally defined later in the article.

summary, we can treat the behavioural rules of agents in the population like Lego bricks. Firstly, if every brick (behavioural rule) used in constructing the process is the same, then we can say something about the entire process by analysing a single brick (a representative agent). Secondly, if our bricks (behavioural rules) are heterogeneous but they all satisfy asymmetry, then we can combine them arbitrarily to construct processes that also satisfy asymmetry.

To give an example, Alice and Bob may update their strategies according to best response rules (Section 4), perhaps occasionally collaborating to play a coalitional best response (Section 6.1). Alice may be a caring person who takes Bob's welfare into account in her decision making (also Section 4). Bob may take his moral philosophy seriously so that his choices have a Kantian (Bergstrom, 1995; Alger and Weibull, 2013, 2016) aspect (Section 6.2). Their friend Colm may follow an imitative rule, perhaps copying the strategy of whichever player currently has the highest payoff (Section 5). For each of these rules, we give conditions under which asymmetry holds. These include conditions on relative incentives such as risk dominance (Harsanyi and Selten, 1988) and an altruistic variant of risk dominance (Maruta, 2002), as well as ordinal conditions such as payoff dominance, maximin and the "Lewis conditions" that relate to a debate between Lewis and Gilbert (1981) over which games are appropriate to the study of conventions. Relevant conditions for classes of behavioural rule are summarized in Figure 2.

Our results suggest that when faced with a problem of conventions, we should first check whether the behavioural rules of agents are asymmetric. For example, if A is risk dominant, then both the logit choice rule and best response with uniform deviations are asymmetric towards A (Section 4). Hence, if some players follow the logit choice rule and the remainder follow best response with uniform deviations, then it follows from Theorems 2 and 3 that the aggregate process is asymmetric. Consequently, the convention at which every player plays A is stochastically stable. We know this without having to consider basins of attraction, transition paths, potential functions, spanning trees or any of the other methodology that usually surrounds such results.[3]

Kreindler and Young (2013), Bilancini and Boncinelli (2020), Newton (2012a), Sawa (2014), Malawski (1989), Schlag (1998), Ellison and Fudenberg (1995), Axelrod (1984), Alós-Ferrer and Schlag (2009), and Ohtsuki *et al.* (2006).

3. Conditions for stochastic stability that apply to a large class of problems and do not require finding transition paths are particularly rare. Exceptions include Blume (1993, 1996), who shows that potential maximizing strategy profiles are stochastically stable when potential games are played under asynchronous log-linear dynamics; Alos-Ferrer and Ania (2005), who provide a sufficient condition for stochastic stability in symmetric games under an imitation process; and Newton and Sawa (2015), who provide a necessary condition for stochastic stability in matching problems. Note that "radius-coradius" methods (Ellison, 2000, citing a no longer extant working paper of Evans) do not fall into this category as they require the calculation of transition paths.

Sections 4–6 consider the asymmetry property under broad classes of behavioural rules. While previous studies have also considered classes of rules (*e.g.* Blume, 2003), our approach stands out with respect to the variety of behavioural rules that it considers. Furthermore, convexity of the set of asymmetric behavioural rules makes a huge number of hybrid rules accessible to study. This is important, as evidence suggests that human behaviour can be a mixed bag, with empirical studies of evolutionary dynamics finding aspects of both best response and imitation (Young and Burke, 2001; Selten and Apesteguia, 2005; Cason *et al.*, 2013; Friedman *et al.*, 2015).[4] Relatedly, classes of rules can be defined by their satisfaction of various properties (Alós-Ferrer and Weidenholzer, 2014). Indeed, the recent empirical work of Nax *et al.* (2016) employs such an approach. Asymmetry can be thought of as such a property. Moreover, as asymmetry is defined directly on choice probabilities, its presence or absence should be empirically observable without knowledge of players' payoffs.

Heterogeneity in various dimensions has been previously considered in the literature on perturbed dynamics. A few examples are (i) the literature on stability in the (generalized) Nash demand game, which incorporates heterogeneity in utility functions and sample sizes in sample-based best response processes (*e.g.* Young, 1993b; Agastya, 1997, 1999; Newton, 2012b); (ii) the literature on "clever agents," which considers best response processes in which some subset of players are "clever" and best respond to conjectured best responses rather than to the current strategy profile (*e.g.* Sáez-Martı and Weibull, 1999; Matros, 2003); (iii) Schipper (2009), which bridges the two most common classes of dynamic process, considering Cournot games played by a population that includes both imitators and best responders; (iv) Alós-Ferrer and Netzer (2015), which considers the effect of heterogeneity in the timing of strategy updating and, more generally, discusses robustness of stochastic stability when the exact updating rules that players follow are unknown. Finally, we note that in the conceptually related, but methodologically different, field of continuous evolutionary dynamics, there exists a literature on dynamics on populations of heterogeneous types (*e.g.* Ely and Sandholm, 2005; Zusai, 2018).

The article is organized as follows. Section 2 gives the model. Section 3 gives our main theoretical results. Section 4 applies these results to payoff-difference based behavioural rules, a class that includes the most popular best response rules. Section 5 does similarly for imitative rules. Section 6 considers coalitional rules, Kantian payoff transformations and altruistic payoff transformations. Section 7 concludes. Proofs are relegated to the Appendix.

2. MODEL

Let V be a finite set of players and $\{A,B\}$ the set of strategies available to each player. Discussion of situations with more than two strategies is deferred to Section 3.1. A strategy profile $\sigma \in \Sigma := \{A,B\}^V$ is a function $\sigma : V \to \{A,B\}$ that associates each player with one of the two strategies. Let σ^A, σ^B be the homogeneous strategy profiles such that for all $i \in V$, $\sigma^A(i) = A$, $\sigma^B(i) = B$. Let σ_S denote σ restricted to the domain $S \subseteq V$. Denote by $V_A(\sigma) \subseteq V$ the set of players who play strategy A at profile σ and by $V_B(\sigma) \subseteq V$ the set of players who play strategy B at profile σ.

Each player $i \in V$ has a payoff function $U_i : \Sigma \to \mathbb{R}$ such that $U_i(\sigma)$ gives the payoff of player i at strategy profile σ. When we consider specific behavioural rules (Section 4 onwards), we shall

4. Of particular relevance to the current study, studies of evolution in coordination games have found support for best response plus deviations with an intentional component (Mäs and Nax, 2016; Lim and Neary, 2016; Hwang *et al.*, 2018).

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assume

$$U_i(\sigma) = \sum_{j\in V\setminus\{i\}} u_{ij}\Big(\sigma(i),\sigma(j)\Big), \qquad \text{(Additive Separability)} \qquad (2.1)$$

where, for all $j \neq i$, $u_{ij}: \{A,B\}^2 \to \mathbb{R}$ gives the payoff of player i from his interaction with player j. If u_{ij} is constant, then the payoff of player i is unaffected by the strategy of player j. In addition, we shall assume

$$u_{ij}(A,A) \geq u_{ij}(B,A),\, u_{ij}(B,B) \geq u_{ij}(A,B). \qquad \text{(Coordination)} \qquad (2.2)$$

A special case of this specification is when each player plays some given coordination game against every other player (*e.g.* Kandori *et al.*, 1993; Young, 1993a) or against some subset of players (*e.g.* Ellison, 2000, 1993).

Assume that the strategy profile evolves according to a discrete time Markov process on Σ. Specifically, we define a family of Markov processes $P = \{P^\varepsilon\}_\varepsilon$ indexed by $\varepsilon \in [0,1)$, where higher values of ε correspond to a greater frequency of perturbations from the *unperturbed process* P^0. Let the state at time t be σ^t. Let P^ε be determined by the following steps. At time $t+1$, select a subset $S \subseteq V$ of updating players according to a probability measure π on the power set of V. Then, let σ^{t+1} be randomly determined according to a probability measure $P^\varepsilon_S(\sigma^t,\cdot)$ satisfying $P^\varepsilon_S(\sigma^t,\sigma) = 0$ if $\sigma_{V\setminus S} \neq \sigma^t_{V\setminus S}$. Note that P^ε_S is also a Markov process on Σ. We shall refer to the family $\{P^\varepsilon_S\}_\varepsilon$ as a *behavioural rule* for S. In summary, the two step strategy updating process selects a set S of updating players before (possibly) updating their strategies, leaving the strategies of players outside of S unchanged. The relationship between P^ε and $\{P^\varepsilon_S\}_{S\subseteq V}$ is given by

$$P^\varepsilon(\sigma,\cdot) = \sum_{S:\pi(S)>0} \pi(S) P^\varepsilon_S(\sigma,\cdot). \qquad (2.3)$$

We consider *regular* behavioural rules (Young, 1993a; see also Sandholm, 2010), the class of rules that satisfy the following conditions. Let P^ε_S be continuous in ε. If $P^0_S(\sigma,\sigma') = 0$ and $P^{\hat{\varepsilon}}_S(\sigma,\sigma') > 0$ for some $\hat{\varepsilon} > 0$, let $\{P^\varepsilon_S(\sigma,\sigma')\}_\varepsilon$ satisfy

$$P^\varepsilon_S(\sigma,\sigma') = \Big(a + o(1)\Big)\varepsilon^k \quad \text{for some } a>0, k>0, \qquad (2.4)$$

where a,k may depend on σ, σ', S, but not on ε; and $o(1)$ represents a term that vanishes as $\varepsilon \to 0$. This class of rules includes popular rules such as the logit choice rule and best response with uniform deviations. Finally, assume that there is strictly positive probability of the players in S retaining their current strategies. That is, $P^\varepsilon_S(\sigma,\sigma) > 0$ for all σ, $\varepsilon \in [0,1)$. Given that the ultimate object of our analysis is the long run behaviour of the process as summarized by its invariant measure, this is without loss of generality.[5]

For $\varepsilon > 0$, assume that any state can be reached with positive probability from any other state in some finite number of steps, therefore the overall process P^ε is irreducible and has a unique invariant probability measure μ^ε on the state space Σ. By standard arguments, the limit

5. If $P^\varepsilon_{S^*}(\sigma^*,\sigma^*) = 0$ for some S^*, σ^*, we can define $\bar{P}^\varepsilon$ such that, for all S such that $\pi(S) > 0$, for all σ, σ', $\sigma \neq \sigma'$, $\bar{P}^\varepsilon_S(\sigma,\sigma) = q + (1-q)P^\varepsilon_S(\sigma,\sigma)$, $\bar{P}^\varepsilon_S(\sigma,\sigma') = (1-q)P^\varepsilon_S(\sigma,\sigma')$, for some arbitrary $q \in (0,1)$, where q can be considered to measure *inertia* added to the original process. $\bar{P}^\varepsilon$ then has the same invariant measure as P^ε, but, for all S such that $\pi(S) > 0$, for all σ, $\bar{P}^\varepsilon_S(\sigma,\sigma) > 0$.

of μ^ε as $\varepsilon \to 0$ exists. For small ε, the process will spend most of the time at states which have positive probability under this limiting measure. These are known as *stochastically stable* states (Foster and Young, 1990).

Definition 1 $\sigma \in \Sigma$ *is* stochastically stable *under* $P = \{P^\varepsilon\}_\varepsilon$ *if* $\lim_{\varepsilon \to 0} \mu^\varepsilon(\sigma) > 0$.

Define the cost $c_S(\sigma,\sigma')$ of a transition by S from σ to σ' as the exponential rate of decay of the probability of such a transition as $\varepsilon \to 0$. That is, for each family of processes $P_S = \{P_S^\varepsilon\}_\varepsilon$, $S \subseteq V$, $\pi(S) > 0$,

$$c_S(\sigma,\sigma') := \begin{cases} \lim_{\varepsilon \to 0} \frac{\log P_S^\varepsilon(\sigma,\sigma')}{\log \varepsilon} & \text{if } P_S^{\hat{\varepsilon}}(\sigma,\sigma') > 0 \text{ for some } \hat{\varepsilon} > 0, \\ \infty & \text{otherwise .} \end{cases} \tag{2.5}$$

Cost functions measure the order of magnitude of transition probabilities for low values of ε. Transitions with a high cost are less likely than transitions with a low cost. From (2.5), we see that if $P_S^0(\sigma,\sigma') > 0$, then $c_S(\sigma,\sigma') = 0$. That is, transitions that can occur under the unperturbed process have zero cost. In contrast, if a transition is only possible for $\varepsilon > 0$, then $c_S(\sigma,\sigma') = k$, where k is the k from expression (2.4).[6] Define a cost function $c(\cdot,\cdot)$ for the overall process P^ε by dropping the S subscripts in (2.5). To relate this to $c_S(\cdot,\cdot)$, observe that if there exist distinct $S,T \subseteq V$ such that $P_S^\varepsilon(\sigma,\sigma') > 0$ and $P_T^\varepsilon(\sigma,\sigma') > 0$, then it is the most likely of these transitions (*i.e.* the lowest cost) which determines the overall likelihood of the transition. Specifically, we derive

Lemma 1 $c(\sigma,\sigma') = \min_{S:\pi(S)>0} c_S(\sigma,\sigma')$.

We consider processes that satisfy a certain type of asymmetry. Asymmetry towards A means, roughly speaking, that if σ, $\tilde{\sigma}$ are such that there is at least as much "A-ness" (players who play A) at $\tilde{\sigma}$ as there is "B-ness" (players who play B) at σ, then switches to strategy A from $\tilde{\sigma}$ are at least as likely as switches to strategy B from σ.

We first give the definition of asymmetry for processes in which a single player updates his strategy (e.g. player i in Figure 3). Given a strategy profile σ, let $\sigma^{(i)}$ denote the strategy profile which is identical to σ except for the strategy of player i. That is, $\sigma^{(i)}(j) = \sigma(j)$ for all $j \neq i$, and $\sigma^{(i)}(i) \neq \sigma(i)$.

Definition 2 $c_{\{i\}}(\cdot,\cdot)$ *is* asymmetric *towards A if, for all* $\sigma,\tilde{\sigma} \in \Sigma$ *such that* $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$*, if* $i \in V_A(\sigma)$ *and* $i \in V_B(\tilde{\sigma})$*, then* $c_{\{i\}}(\sigma,\sigma^{(i)}) \geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$.

Asymmetry towards B can be defined by a simple relabelling of strategies. For the remainder of the article, we shall assume that unless specified otherwise, "asymmetry" refers to asymmetry towards A.

Asymmetry was originally defined by Peski (2010) for $c(\cdot,\cdot)$, that is for the aggregate process. Taking σ and $\tilde{\sigma}$ as in Definition 2, his definition requires that for a transition away from σ to any given σ', we can find a transition which is at least as likely from $\tilde{\sigma}$ to some $\tilde{\sigma}'$, such that the latter

6. To illustrate, a transition with a probability of ε^2 has a cost of 2, whereas a transition with the lower probability of ε^3 has a cost of 3. The most common interpretation in the literature for these powers of ε has been the number of mutations required to effect a transition. This corresponds to the best response with uniform deviations rule that we consider in Section 4.

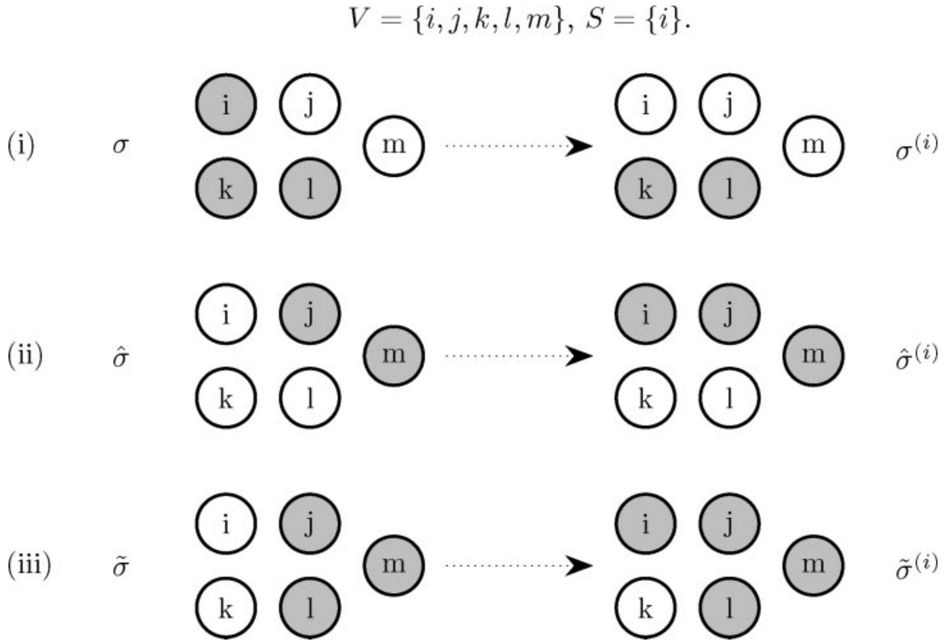

FIGURE 3

Asymmetry at the individual level. Vertices shaded grey play A. Unshaded vertices play B. Note that $V_B(\sigma) = V_A(\hat{\sigma}) \subseteq V_A(\tilde{\sigma})$. That is, all players who play B at σ play A at $\hat{\sigma}$ and $\tilde{\sigma}$. In words, there is at least as much "A-ness" at $\hat{\sigma}$ and $\tilde{\sigma}$ as there is "B-ness" at σ. Consequently, asymmetry (Definition 2) implies that a transition from σ to $\sigma^{(i)}$ (Panel [i]) is weakly less likely than a transition from $\hat{\sigma}$ to $\hat{\sigma}^{(i)}$ (Panel [ii]) or a transition from $\tilde{\sigma}$ to $\tilde{\sigma}^{(i)}$ (Panel [iii]).

transition only involves switches to A and the set of players who play A at $\tilde{\sigma}'$ includes the set of players who play B at σ'. This definition can also be applied to arbitrary, possibly non-singleton S. We show in Appendix B that for singleton S, Definitions 2 and 3 are equivalent.

Definition 3 *$c_S(\cdot,\cdot)$ is* asymmetric *towards A if, for any $\sigma, \sigma', \tilde{\sigma} \in \Sigma$, such that $V_B(\sigma) \subseteq V_A(\tilde{\sigma})$, there exists $\tilde{\sigma}' \in \Sigma$ such that $V_A(\tilde{\sigma}) \subseteq V_A(\tilde{\sigma}')$, $V_B(\sigma') \subseteq V_A(\tilde{\sigma}')$ and $c_S(\sigma, \sigma') \geq c_S(\tilde{\sigma}, \tilde{\sigma}')$.*

The concept of asymmetry can be extended from cost functions to underlying processes in the obvious way.

Definition 4 *$P_S = \{P_S^\varepsilon\}_\varepsilon$ is* asymmetric *(towards A) if its cost function is asymmetric (towards A).*

It turns out that asymmetry of the aggregate process is a sufficient condition for the stochastic stability of σ^A.

Theorem P (Peski, 2010). *If $c(\cdot,\cdot)$ is asymmetric towards A, then σ^A is stochastically stable.*[7]

7. Peski (2010) also gives a strict version of asymmetry that ensures that σ^A is *uniquely* stochastically stable. The theorems in the current paper can be stated and proved for strict asymmetry (see Newton, 2019). We choose to present results for (non-strict) asymmetry as the definition is cleaner and it admits broader classes of behavioural rules.

In the cited paper, Theorem P is used to show that risk dominance of strategy A implies stochastic stability of σ^A under best response with either uniform or payoff-dependent deviations for any network of interaction.[8] In the next section, we give results that allow us to apply Theorem P to processes that admit a great deal of heterogeneity in behavioural rules.

3. COMBINING ASYMMETRIES

We now present a lemma upon which the theorems of this section build. Asymmetry of cost functions is preserved under minima.

Lemma 2 *If cost functions c_1 and c_2 are asymmetric towards A, then* $\min\{c_1, c_2\}$ *is also asymmetric towards A.*

It follows from Lemma 2 that if we combine two asymmetric processes in such a way that the resulting process has a cost function which is a minimum of the two original cost functions, then the resulting process will also be asymmetric. This result allows us to consider two types of heterogeneity in behavioural rules. These are heterogeneity within agents (Alice sometimes follows one rule and sometimes follows another rule) and heterogeneity between agents (Alice and Bob follow different rules). The first of these considers a set S of updating players that sometimes chooses according to one rule and sometimes according to another.

Theorem 1 (Heterogeneity within agents)
If $\tilde{P}_S$ and $\bar{P}_S$ are asymmetric towards A, then P_S defined by $P_S^\varepsilon = \lambda \tilde{P}_S^\varepsilon + (1-\lambda)\bar{P}_S^\varepsilon$, $\lambda \in (0,1)$, is asymmetric towards A.

For example, Alice may sometimes follow a best response rule (see Section 4) and sometimes follow an imitative rule (see Section 5), but as long as both rules are asymmetric, Theorem 1 tells us that a process which combines them will also be asymmetric.[9,10]

The next theorem considers heterogeneity across updating sets of players. Each S that updates with positive probability may do so according to a different behavioural rule.

Theorem 2 (Heterogeneity between agents)
If P_S is asymmetric towards A for all $S \subseteq V$ such that $\pi(S) > 0$, then P is asymmetric towards A.

For example, Alice may try to maximize Bob's payoff (see Section 4), Bob may follow Homo Moralis preferences (see Section 6.2), and sometimes Alice and Bob may even form a coalition

8. To give some context, when strategy A is risk dominant, stochastic stability of σ^A under best response plus uniform deviations under uniform interaction (*i.e.* u_{ij} independent of i, j) was proved by Young (1993a) and Kandori *et al.* (1993); possible multiplicity of stochastically stable states under best response plus uniform deviations for general networks of interaction is described in Blume (1996); stochastic stability of σ^A under logit choice (a form of best response plus payoff-dependent deviations) for general finite networks of interaction is described in Blume (1996) and Young (1998).

9. Note that the examples following Theorems 1, 2, and 3 are simple and illustrative. More complicated examples are easy to construct. For example, Alice might follow one rule when she updates at the same time as Bob and another rule when she updates at the same time as Colm. Alternatively, it could be that when Alice and Bob update at the same time, their rules are perfectly correlated so that exactly one of them follows an imitative rule and exactly one of them follows a best response rule.

10. A number of papers in the literature construct behavioural rules by additively combining perturbations with an unperturbed behavioural rule. Supplementary Appendix H discusses how Theorem 1 applies to such models.

for their mutual benefit (see Section 6.1), but as long as all three rules are asymmetric, Theorem 2 tells us that the aggregate process will also be asymmetric. As well as heterogeneity, Theorem 2 can help us to understand homogeneity. Consider a situation in which the only S selected with positive probability are singleton players, each of whom follows the same behavioural rule. If we can give some general conditions under which this behavioural rule is asymmetric for any given *representative agent*, then Theorem 2 implies that the aggregate process must be asymmetric under the same conditions. Later in the paper (Corollary 1), we show that the most famous results in this literature can be recovered by this method.

The next theorem allows us to consider disjoint sets of players S and T that simultaneously and independently follow asymmetric behavioural rules. When this is the case, the joint behavioural rule for $S \cup T$ is also asymmetric.

Theorem 3 (Heterogeneity in timing)
Let $S, T \subseteq V$, $S \cap T = \emptyset$, P_S and P_T be asymmetric towards A. If $P_{S\cup T}$ satisfies, for all ε, σ, $\sigma'_{S\cup T}$,

$$P^{\varepsilon}_{S\cup T}\left(\sigma,(\sigma'_{S\cup T},\sigma_{V\setminus(S\cup T)})\right)=P^{\varepsilon}_{S}\left(\sigma,(\sigma'_{S},\sigma_{V\setminus S})\right)P^{\varepsilon}_{T}\left(\sigma,(\sigma'_{T},\sigma_{V\setminus T})\right),$$

then $P_{S\cup T}$ is asymmetric towards A.

For example, if Alice follows an asymmetric behavioural rule and Bob follows an asymmetric behavioural rule, then the aggregation of these rules is asymmetric regardless of whether Alice and Bob adjust their strategies at different times or at the same time. In general, the possibility of simultaneous strategic updating can be important. Alós-Ferrer and Netzer (2015) define a robustness concept based on the possibility of the identity of stochastically stable states being affected by simultaneity in updating. Arieli and Young (2016) need a particular combination of simultaneity and non-simultaneity in strategy updating in order to obtain rapid convergence to Nash equilibrium in a class of learning models. Theorem 3 shows that, when it comes to asymmetry, we do not have to worry.

3.1. *Asymmetry with more than two strategies*

Before we move on to discuss examples of asymmetric processes and therefore the scope of the above theorems, we briefly discuss asymmetry in environments with more than two strategies. Indeed Peski (2010) defines asymmetry and proves Theorem P for such environments. The fundamental difference is that with two strategies we consider asymmetry in terms of strategy A versus strategy B, whereas with more than two strategies we consider asymmetry in terms of strategy A versus all strategies other than A. Specifically, denote by $V_{\neg A}(\sigma) \subseteq V$ the set of players who play any strategy other than strategy A at profile σ. Asymmetry towards A is then defined by replacing $V_B(\cdot)$ in Definition 3 by $V_{\neg A}(\cdot)$. All of the above theorems and their proofs continue to hold after this substitution.

Further discussion, a formal definition of asymmetry with arbitrary finite strategy sets and an example are provided in Supplementary Appendix I. For the remainder of the main body of the article, we focus on the two strategy case under a wide variety of behavioural rules.

4. CHOICE BASED ON PAYOFF DIFFERENCES

We first consider behavioural rules according to which the probability of an individual player switching from his current strategy to the alternative strategy decreases in the vector of payoff losses from the switch. An updating player following such a rule acts according to a predisposition to improve things, or at least not make them worse, for some group of players. When this group is the updating player himself, we have the subclass of best/better response rules.[11] As we shall see, this class includes many rules that have been considered in the literature.

4.1. *Definition of payoff-difference based rules*

Consider the vector of differences in payoff for every player when player i changes his strategy so that the strategy profile changes from σ to $\sigma^{(i)}$. That is, consider

$$D_i^\sigma := \left(U_j(\sigma) - U_j(\sigma^{(i)})\right)_{j\in V} \in \mathbb{R}^V.$$

Positive elements of D_i^σ correspond to payoff losses and negative elements of D_i^σ correspond to payoff gains.

Let a *payoff-difference based* rule for player i be a behavioural rule that gives the following cost function. For non-decreasing $\Upsilon_i(\cdot):\mathbb{R}^V \to \mathbb{R}_+$,

$$c_{\{i\}}(\sigma,\sigma') := \begin{cases} 0 & \text{if } \sigma' = \sigma, \\ \Upsilon(D_i^\sigma) & \text{if } \sigma' = \sigma^{(i)}, \\ \infty & \text{otherwise.} \end{cases} \tag{4.1}$$

For completeness, in Appendix C, we give explicit choice probabilities that imply such cost functions. Such rules satisfy the restriction on behaviour that if a transition is at least as good (measured by changes in payoff) for everybody as another transition, then the first transition should be no less likely to occur than the second. We shall now illustrate the breadth and flexibility of this class of rules by giving some examples. Following this, we give sufficient conditions for the asymmetry of such processes.

4.2. *Examples of payoff-difference based rules*

4.2.1. Utilitarian rules. A player i follows a *utilitarian* rule if, for some non-negative vector $\lambda \in \mathbb{R}_+^V$, we have that

$$\Upsilon_i(x) = [\lambda \cdot x]_+, \tag{4.2}$$

where $[x]_+ = \max\{0,x\}$. Under this rule, the probability of player i changing his strategy is decreasing in a weighted sum of payoff changes when he does so. A special case of this is when $\lambda_i = 1$ and $\lambda_j = 0$ for $j \neq i$, in which case we have *best response with log-linear deviations*, which for small ε approximates the logit choice rule (see Blume, 1993; Alós-Ferrer and Netzer, 2010). This rule is self-regarding in the following sense.

Definition 5 *A rule Υ_i is* self-regarding *if $\Upsilon_i(x) = f(x_i)$ for some non-decreasing function f : $\mathbb{R} \to \mathbb{R}_+$.*

11. Fixed points of such rules define the equilibrium concepts of Cournot (1838) and Nash (1950). In his proofs of the existence of Nash equilibria, Nash uses two best/better response mappings. Most famously (Nash, 1950), the classic best response correspondence that is definitional to Nash equilibrium, but also, in an alternative proof (Nash, 1951), a smoothed better response correspondence that allows the use of Brouwer's rather than Kakutani's fixed point theorem.

The class of self-regarding payoff-difference based rules is effectively the class of *skew-symmetric* rules considered by Blume (2003) and Norman (2009a). In contrast, if $\lambda_j = 1$ for some $j \neq i$ and $\lambda_k = 0$ for $k \neq j$, then we have a *best friend forever* rule, where player i makes his decisions according to their impact on player j. Clearly, this rule is not self-regarding.

4.2.2. Best response with uniform deviations. A player i follows *best response with uniform deviations* (Kandori *et al.*, 1993; Young, 1993a) if

$$\Upsilon_i(x) = \begin{cases} 1 & \text{if } x_i > 0 \\ 0 & \text{if } x_i \leq 0 \end{cases}, \tag{4.3}$$

so that, for small ε, player i will rarely change his strategy unless his payoff weakly increases as a consequence.

In Supplementary Appendix E, several further examples of payoff-difference based rules are given. These include *best response with log-quadratic deviations*, which for small ε approximates the probit choice rule in two strategy environments (Dokumaci and Sandholm, 2011); *Hippocratic* rules which depend only on payoff losses and disregard payoff gains; and best response rules with *switching costs* (Norman, 2009b).

4.3. *Asymmetry of payoff-difference based rules*

Recall that a risk dominant strategy (Harsanyi and Selten, 1988) for player i is a strategy that maximizes his payoff when he faces an opponent who plays each strategy with equal probability. Similarly, we define an altruistically risk dominant strategy for player i against player j to be a strategy that player i should play to maximize the payoff of player j when player j plays each strategy with equal probability. Maruta (2002) refers to this latter condition as dominance with respect to homogeneous externality, as it compares the change in payoff of players of each strategy when an opponent switches to that strategy. However, an interpretation as altruistic risk dominance emphasizes the symmetry with risk dominance that is important to the results of this section.

Definition 6 *Strategy A is RD_i (risk dominant for i) if*

$$\sum_{j \in V \setminus \{i\}} u_{ij}(A,A) + u_{ij}(A,B) \geq \sum_{j \in V \setminus \{i\}} u_{ij}(B,A) + u_{ij}(B,B);$$

and ARD_{ij} (altruistically risk dominant for i against j) if

$$u_{ji}(A,A) + u_{ji}(B,A) \geq u_{ji}(A,B) + u_{ji}(B,B).$$

These two properties turn out to be exactly what is required to give asymmetry of payoff-difference based rules. Considering a population of only Alice and Bob, the intuition is clear. When A is risk dominant for Alice, the payoff advantage to her of playing A (rather than B) when Bob plays A is greater than the payoff advantage to her of playing B (rather than A) when Bob plays B. A similar logic applies if we consider altruistic risk dominance and the effect of Alice's strategy on Bob's payoffs. These asymmetries in payoff differences then translate directly into asymmetry of choice probabilities.[12]

12. Much prior literature considers a given coordination game played across pairs on a network. Even allowing for directed and weighted networks, this restricts each u_{ij} to the linear form $u_{ij}(\sigma(i),\sigma(j)) = \lambda_{ij} u(\sigma(i),\sigma(j))$ for some $\lambda_{ij} \in \mathbb{R}_+$.

Proposition 1 *If player i follows a payoff-difference based rule, A is RD$_i$ and*

(i) Υ_i *is self-regarding, or*
(ii) A is ARD$_{ij}$ for all j,

then $c_{\{i\}}(\cdot,\cdot)$ *is asymmetric towards A.*

So, if Alice follows a self-regarding payoff-difference based rule such as best response with uniform deviations and A is risk dominant for Alice, then her cost function will be asymmetric (Proposition 1[i]). If Bob follows a utilitarian rule and tries to maximize the total payoff for him and Alice, A is risk dominant for Bob and altruistically risk dominant for Bob against Alice, then his cost function will be asymmetric (Proposition 1[ii]). If Alice and Bob alter their strategies simultaneously, then the resulting cost function for $S=\{\text{Alice},\text{Bob}\}$ will also be asymmetric (Theorem 3). If P^{ε} is such that sometimes Alice alters her strategy, sometimes Bob alters his strategy and sometimes they alter their strategies simultaneously, then the resulting cost function for the combined process is asymmetric (Theorem 2), hence Theorem P applies and the state at which both Alice and Bob play A is stochastically stable.

4.4. *Summary of the proof technique*

Proposition 1 and subsequent propositions in the main body of the article are proven by considering asymmetry as the implication of two simpler conditions. These conditions are formally defined in Appendix B, but can be simply described here in words. The first condition considers strategy profiles σ and $\hat{\sigma}$ that are complete opposites, such that for all $i\in V$, $\sigma(i)\neq\hat{\sigma}(i)$. Such profiles are illustrated in Figure 3. When transitions towards A from $\hat{\sigma}$ are at least as likely as corresponding transitions towards B from σ, we say that the process is *weakly asymmetric* towards A. The second condition considers strategy profiles $\hat{\sigma}$ and $\tilde{\sigma}$ (also illustrated in Figure 3) such that every player who plays A at $\hat{\sigma}$ also plays A at $\tilde{\sigma}$. When transitions towards A from $\tilde{\sigma}$ are at least as likely as corresponding transitions towards A from $\hat{\sigma}$, we say that the process is *supermodular* towards A. Taken together, weak asymmetry and supermodularity are sufficient conditions for asymmetry (Lemma B.2 in Appendix B). The proof of Proposition 1 in Appendix C proceeds by proving weak asymmetry and supermodularity and thus asymmetry.

4.5. *Conditions are sufficient but not necessary for asymmetry*

The conditions in Proposition 1 are sufficient for the asymmetry of all payoff-difference based rules. However, for any particular given payoff-difference based rule, weaker conditions will usually suffice. Consider $V=\{i,j\}$ and let i follow a utilitarian rule (see Section 4.2) with $\lambda_i=\lambda_j=1/2$. That is, i makes decisions according to the average effect it has on himself and on player j. Let payoffs be given by the game in Figure 4. Note that $u_{ij}(A,A)+u_{ij}(A,B)=7<8=u_{ij}(B,A)+u_{ij}(B,B)$, so that A is not risk dominant for i. From any strategy profile σ at which player i plays A, the average payoff of the two players will decrease if i switches to B. From (4.1) and (4.2), we obtain that $c_{\{i\}}(\sigma,\sigma^{(i)})>0$. However, from any strategy profile $\tilde{\sigma}$ at which player i plays B, the average payoff of the two players will increase if i switches to A. Therefore,

When this is the case, $u(A,A)+u(A,B)\geq u(B,A)+u(B,B)$ implies that A is RD$_i$ for all i. Similarly, $u(A,A)+u(B,A)\geq u(A,B)+u(B,B)$ implies that A is ARD$_{ij}$ for all i,j. Maximin and payoff dominance conditions considered later in the article also simplify under these payoffs.

		Player j	
		A	B
Player i	A	7,7	0,6
	B	6,0	2,2

FIGURE 4

Payoffs. For each combination of A and B, entries give payoffs for player i and j respectively.

$c(\tilde{\sigma},\tilde{\sigma}^{(i)})=0$. Clearly, the conditions of Definition 2 will be satisfied, so that the behavioural rule of player i is asymmetric towards A despite not satisfying the conditions of Proposition 1.

4.6. *Relation to the literature*

If A is RD_i and player i follows a self-regarding payoff-difference based rule, then $c_{\{i\}}$ is asymmetric towards A (Proposition 1[i]). If this holds for all $i\in V$ and only individual players update their strategies, then the aggregate process is asymmetric towards A (Theorem 2), hence Theorem P applies and σ^A is stochastically stable. We have the following corollary.

Corollary 1 *Let $\pi(\{i\})>0$ for all $i\in V$, $\pi(S)=0$ otherwise. If, for all $i\in V$, A is RD_i and i follows a self-regarding payoff-difference based rule, then σ^A is stochastically stable.*

This corollary nests existing results on stochastic stability under best response with uniform deviations and own-payoff based rules (Peski, 2010, Theorems 2 and 3, respectively), special cases of which include best response with uniform deviations and uniform interaction (Kandori *et al.*, 1993; Young, 1993a); best response with uniform deviations on specific interaction structures such as the ring network and the two dimensional square lattice with von Neumann neighbourhoods (Ellison, 2000, 1993); and best response with log-linear deviations for any interaction structure (Blume, 1996; Young, 1998). Considering the full set of self-regarding payoff-difference based rules, but restricting attention to uniform interaction with each agent following the same rule, the Corollary is effectively the result of Blume (2003, Theorem 1). Combining this with Theorem 3 of the current article then gives us the equivalent result for simultaneous choice Norman (2009a, Theorem 1).[13]

5. IMITATIVE CHOICE

A process is imitative if an updating player is more likely to switch to a strategy that currently obtains high payoffs for those who play it. Let $C\subseteq V$ be player i's *comparison set*. When player i considers changing his strategy, his switching probability will depend on the current payoffs of the players in his comparison set. Specifically, his switching probability is weakly decreasing in the payoffs of those in his comparison set who currently play the same strategy as himself, and weakly increasing in the payoffs of those in his comparison set who play the alternative strategy.[14] Here, we specifically focus on two rules, although the reader interested in further

13. The qualifier "effectively" here refers to the fact that both Blume (2003) and Norman (2009a) deal with strict risk dominance and unique stochastic stability for large populations, whereas here we deal with (not necessarily strict) risk dominance and (not necessarily unique) stochastic stability, without any restriction on population size.

14. A variety of imitative rules have been studied in the literature. For example, player i may sample some player j in his comparison set and adopt j's strategy if j obtains a higher payoff than i, that is if $U_j(\sigma)>U_i(\sigma)$ (Malawski, 1989).

study of imitative rules will find parts of the proofs in Supplementary Appendix D (particularly Lemma 7) to be of general applicability.

5.1. *Condition dependence*

We begin with the simple case in which player i has a comparison set that contains only himself. That is, $C=\{i\}$. In this case, the switching probability for player i decreases in his current payoff $U_i(\sigma)$ and is independent of the payoffs of the other players. More formally, from any state σ, the transition cost $c_{\{i\}}(\sigma,\sigma^{(i)})$ is given by some weakly increasing function that has the current payoff of player i as its argument. This is known as *condition dependence* (Bilancini and Boncinelli, 2020) after the biology literature. The justification for the use of such a rule is simple: if one is obtaining a low payoff, it makes sense to try something else.

A strategy is payoff dominant for i against j if, when i and j coordinate on that strategy, a higher payoff is obtained for i than would be the case if they coordinated on the other strategy. Similarly, a strategy is maximin for i against j if it maximizes the payoff of i in the worst case scenario, that of miscoordination.

Definition 7 *Strategy A is PD_{ij} (payoff dominant for i against j) if*

$$u_{ij}(A,A)\geq u_{ij}(B,B);$$

and MM_{ij} (maximin for i against j) if

$$u_{ij}(A,B)\geq u_{ij}(B,A).$$

These two properties turn out to be exactly what is required to give asymmetry of condition dependent rules. The intuition is clear if we consider the case of only two players, $V=\{i,j\}$. Payoff dominance of A for i against j ensures that, comparing two states of coordination $\sigma=(\sigma_i,\sigma_j)=(A,A)$ and $\tilde{\sigma}=(B,B)$, we have $U_i(\sigma)\geq U_i(\tilde{\sigma})$, so switches by i to A at $\tilde{\sigma}$ are at least as likely as switches by i to B at σ. Similarly A being maximin for i against j ensures that, comparing states of miscoordination $\sigma=(A,B)$ and $\tilde{\sigma}=(B,A)$, switches by i to A at $\tilde{\sigma}$ are at least as likely as switches by i to B at σ. Finally, coordination ensures that the same applies for $\sigma=(A,A)$ and $\tilde{\sigma}=(B,A)$.

Proposition 2 *If player i follows a condition dependent rule, A is PD_{ij} and MM_{ij} for all j, then $c_{\{i\}}(\cdot,\cdot)$ is asymmetric towards A.*

Again, the conditions of Proposition 2 are sufficient but not necessary. Their tightness will depend on the situation under consideration. Consider $V=\{i,j\}$ and let i follow a condition dependent rule such that $c_{\{i\}}(\sigma,\sigma^{(i)})=0$ if $U_i(\sigma)<0$ and $c_{\{i\}}(\sigma,\sigma^{(i)})=1$ if $U_i(\sigma)\geq 0$. This can be interpreted as player i switching his strategy with a high probability if his payoff is less than some

A smoothed version of this rule has i switching to j's strategy with a probability proportional to $U_j(\sigma)-U_i(\sigma)$ (Schlag, 1998). Alternatively, player i may simultaneously consider the payoffs of all of the players in his comparison set and adopt the strategy associated with the highest average payoff (Ellison and Fudenberg, 1995) or the strategy of whichever player currently obtains the highest payoff (Axelrod, 1984). In the biology literature (see Ohtsuki *et al.*, 2006), it is common to assume that the strategy of each player in the comparison set is adopted with a probability proportional to that player's payof—a *death–birth Moran* process. If every player simultaneously follows such a process (a possible application of Theorem 3), then we have a *Wright–Fisher* process (see Lehmann *et al.*, 2007). For a survey of imitative rules, the reader is referred to Alós-Ferrer and Schlag (2009).

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(i) Player i (rows) vs. Player j (columns)

	A	B
A	3	−1
B	1	2

(ii) Player i (rows) vs. Player j (columns)

	A	B
A	3	−3
B	−2	−1

FIGURE 5

Payoffs. For each combination of A and B, entries give payoffs for player i.

threshold (zero in this case) and rarely switching his strategy otherwise. Neither of the possible sets of payoffs for player i that are given in Figure 5 satisfy A being maximin for i against j. Considering the payoffs in Figure 5(i), we see that for $\sigma = (\sigma_i, \sigma_j) = (A, B)$, $\tilde{\sigma} = (B, A)$, we have $c_{\{i\}}(\sigma, \sigma^{(i)}) = 0$ and $c_{\{i\}}(\tilde{\sigma}, \tilde{\sigma}^{(i)}) = 1$, violating the conditions for asymmetry towards A (Definition 2). However, if instead payoffs are as given by Figure 5(ii), then any switch by player i from B to A has a cost of zero regardless of the strategy of player j. Thus the conditions for asymmetry towards A are trivially satisfied (see Definition 2), despite the conditions of Proposition 2 not being satisfied.

5.2. *Imitate the best*

We now consider *imitate-the-best* rules. When a player follows such a rule, he considers the players in his comparison set and compares the highest payoff obtained by any player who plays A to the highest payoff obtained by any player who plays B. He is then more likely to switch to strategies associated with high payoffs and less likely to switch away from strategies associated with high payoffs. More formally, from any state σ, the transition cost $c_{\{i\}}(\sigma, \sigma^{(i)})$ is given by some function that has two arguments. The first argument is the maximum payoff obtained at σ among all players in player i's comparison set who play the same strategy as i at σ. The function weakly increases in the first argument. The second argument is the maximum payoff obtained at σ among all players in player i's comparison set who play the alternative strategy at σ. The function weakly decreases in the second argument. If one or the other of these maxima is undefined due to no players in the comparison set playing that strategy, then $c_{\{i\}}(\sigma, \sigma^{(i)})$ takes its lowest and highest values respectively. Details can be found in Supplementary Appendix D. Conditions that guarantee asymmetry for this class of behavioural rules are similar to those discussed above for condition dependence, the difference being that when following an imitate-the-best rule, the choices of player i depend on the payoffs of other players in his comparison set.

Proposition 3 *If player i follows an imitate-the-best rule, A is* PD_{jk} *and* MM_{jk} *for all* $j \in C$, k, *then* $c_{\{i\}}(\cdot, \cdot)$ *is asymmetric towards A.*

Again, the conditions of Proposition 3 are sufficient but not necessary. To see this, note that if $C = \{i\}$, then imitate-the-best rules reduce to condition dependent rules, which we have already discussed. We further note that conditions PD_{jk} and MM_{jk} are stronger than conditions on payoffs in prior studies (Robson and Vega-Redondo, 1996; Alós-Ferrer and Weidenholzer, 2008; Khan, 2014) that give stochastic stability of σ^A under an imitate-the-best rule when A is PD_{jk} but not MM_{jk}. These studies involve interaction structures and comparison sets set up in such a way that the payoff advantage of A over B from coordinated pairs (as discussed before Proposition 2) outweighs any payoff disadvantage from miscoordinated pairs.

The proofs of Propositions 2 and 3 use the fact that PD_{ij} and MM_{ij} together imply $u_{ij}(B,B) \geq u_{ij}(B,A)$ and $u_{ij}(A,A) \geq u_{ij}(A,B)$, meaning that switching a player j from B to A increases the payoff of i if he is already playing A and decreases the payoff of i if he is already playing B. For condition dependent and imitate-the-best rules, this in turn increases the probability of player i choosing A.[15] Note that this argument would not hold if we instead considered an imitative process based on average (rather than best) payoffs, as switching a player from B to A can lead to a reduction in the average payoff of players who play A and an increase in the average payoff of players who play B.

6. COALITIONS AND KANTIANISM

6.1. *Coalitional choice*

In Supplementary Appendix F, we consider a coalitional variant of payoff-difference based rules. This models situations in which subsets of players get together and decide whether they should play A or play B (see Newton, 2012a; Sawa, 2014; Newton and Angus, 2015). When it comes to conditions for asymmetry, the difference between individualistic and coalitional payoff-difference based rules can be concisely explained. Firstly, similar to the individualistic case, we must consider the interaction of a coalition of players with those outside of the coalition. This gives rise to risk dominance and altruistic risk dominance conditions similar to those we saw in Section 4. Secondly, unlike the individualistic case, we must also consider the payoffs that players within the coalition obtain from interacting with each other. This gives rise to payoff dominance conditions similar to those we saw in Section 5.

6.2. *Payoff transformations—Kantian*

Sometimes payoff transformations can carry conceptual weight.[16] In Supplementary Appendix G, we consider the *Homo Moralis* transformation (Bergstrom, 1995; Alger and Weibull, 2013, 2016) which combines standard payoffs with a *Kantian* component, the latter being a consideration along the lines of "what strategy should I play if my opponent plays the same as I do?" When the behavioural rule is payoff-difference based and self-regarding, this leads to sufficient conditions for asymmetry that are a convex combination of risk dominance and payoff dominance, the payoff dominance aspect receiving higher weights when the Kantian component of behaviour is stronger.[17]

6.3. *Payoff transformations—altruistic*

In Section 4, we considered other-regarding behavioural rules and considered their relationship with altruistic risk dominance. A less flexible, though common approach is to apply a transformation that combines a player's own payoffs with those of his opponents. We show in

15. Interestingly, these inequalities correspond to conditions that Lewis (1969) imposes on the games he considers in his philosophical theory of conventions. Gilbert (1981) later argued that these conditions were too stringent. Indeed, as we saw in Section 4, they are not directly relevant to the class of payoff-difference based rules that has predominated in game theoretic forays into this territory. However, they are of direct relevance to imitative choice.

16. Recent work has considered such transformations in relation to risk attitudes and loss aversion (Sawa and Wu, 2018; Nax and Newton, 2019).

17. Alger and Weibull (2016) show that a stronger Kantian component of behaviour is more likely to evolve in environments where interaction displays high levels of positive assortativity. Assortativity in turn can then be subject to evolutionary pressures (see Nax and Rigos, 2016; Newton, 2017; Wu, 2017).

Supplementary Appendix G that such an approach also fits into our framework and that, perhaps unsurprisingly, it leads to sufficient conditions for asymmetry that are a convex combination of risk dominance and altruistic risk dominance.

7. DISCUSSION

7.1. *Bias towards A does not guarantee stochastic stability of σ^A*

Theorem 2 together with Theorem P tells us that if all agents' behavioural rules are asymmetric towards A, then σ^A is stochastically stable. However, not every bias towards strategy A guarantees stochastic stability of σ^A. In Section 4.4, we described how asymmetry can be considered as an implication of two other conditions, weak asymmetry and supermodularity, both formally defined in Appendix B. As the name suggests, weak asymmetry is a weaker condition than asymmetry. Like asymmetry towards A, weak asymmetry towards A can be considered as a bias towards A. However, even if every agent follows a weakly asymmetric rule, it may still be the case that σ^A is not stochastically stable. In Appendix B, we show this with an example in which a bias towards A coexists with a bias towards playing a different strategy to one's opponent. When the former bias is weaker than the latter bias, the only stochastically stable strategy profiles may involve some players playing A and other players playing B.

7.2. *Non-asymmetry preserving combinations of rules*

We have seen how behavioural rules can be combined while retaining asymmetry. We now briefly discuss a form of combination to which asymmetry is not robust. Consider rules such that an agent will have a high proclivity to play A when some conditions are satisfied, and will have a low proclivity to play A when the conditions are not satisfied. Consider two such "parent" rules and assume they are asymmetric towards A. By Theorem 1, any compound rule that plays the first rule some of the time and the second rule the rest of the time is asymmetric towards A. In contrast, consider a "child" rule under which an agent will have a high proclivity to play A when the conditions of both parent rules are satisfied, and will have a low proclivity to play A otherwise. Clearly, the child rule is less disposed towards A than either of the parent rules. If this effect is strong enough, then the child rule will not be asymmetric towards A.[18]

7.3. *Afterword*

In the first part of this article (Section 3), we showed how behavioural rules can be combined whilst retaining asymmetry. We considered heterogeneity within agents' rules (Theorem 1), heterogeneity between agents' rules (Theorem 2) and heterogeneity in the timing of strategy updating (Theorem 3). In the second part of the article (Sections 4–6), we discussed behavioural rules to which our theorems apply. It will be apparent to the reader that this does not exhaust what can be said on this subject. Important avenues for future research would seem to include (i) the study of more behavioural rules; (ii) the study of different payoff specifications; (iii) empirically

18. For example, consider $V=\{i,j\}$ and rules for player i. Let σ be the current strategy profile. **Parent rule 1:** If $\sigma_i=A$ (respectively B), then player i chooses A with probability $1-\varepsilon^2$ (respectively ε). **Parent rule 2:** If $\sigma_j=A$ (respectively B), then player i chooses A with probability $1-\varepsilon^2$ (respectively ε). It is simple to verify that these rules are asymmetric towards A. **Child rule:** If $\sigma_i=A$ and $\sigma_j=A$, then player i chooses A with probability $1-\varepsilon^2$, otherwise he plays A with probability ε. Considering $\sigma_i=A$, $\sigma_j=B$, $\tilde{\sigma}_i=B$, $\tilde{\sigma}_j=A$, we have that $c(\sigma,\sigma^{(i)})=0<1=c(\tilde{\sigma},\tilde{\sigma}^{(i)})$, violating the condition for asymmetry in Definition 2.

testing observed behaviour for asymmetry; (iv) applications to specific economic problems that admit heterogeneity in behaviour.

Appendix

A. PROOFS OF GENERAL RESULTS

Proof of Lemma 1. Let $\frac{\log 0}{\log\varepsilon}:=\infty$. For all $\varepsilon\in(0,1)$, from (2.3), we obtain

$$\max_{S:\pi(S)>0}\pi(S)P^{\varepsilon}_{S}(\sigma,\sigma')\le P^{\varepsilon}(\sigma,\sigma')\le\max_{S:\pi(S)>0}P^{\varepsilon}_{S}(\sigma,\sigma'). \tag{A.1}$$

As $\log\varepsilon<0$, we have that $\frac{\log x}{\log\varepsilon}$ is decreasing in x. Applying this transformation to (A.1), we obtain

$$\frac{\log\max_{S:\pi(S)>0}\pi(S)P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}\ge\frac{\log P^{\varepsilon}(\sigma,\sigma')}{\log\varepsilon}\ge\frac{\log\max_{S:\pi(S)>0}P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}, \tag{A.2}$$

which rearranges to give

$$\min_{S:\pi(S)>0}\frac{\log\pi(S)P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}\ge\frac{\log P^{\varepsilon}(\sigma,\sigma')}{\log\varepsilon}\ge\min_{S:\pi(S)>0}\frac{\log P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}. \tag{A.3}$$

The first expression in (A.3) is bounded as follows

$$\max_{S:\pi(S)>0}\frac{\log\pi(S)}{\log\varepsilon}+\min_{S:\pi(S)>0}\frac{\log P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}\ge\min_{S:\pi(S)>0}\frac{\log\pi(S)P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}. \tag{A.4}$$

Combining (A.3) and (A.4), we have

$$\underbrace{\max_{S:\pi(S)>0}\frac{\log\pi(S)}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0}+\min_{S:\pi(S)>0}\underbrace{\frac{\log P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to c_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}\ge\underbrace{\frac{\log P^{\varepsilon}(\sigma,\sigma')}{\log\varepsilon}}_{\to c(\sigma,\sigma')\text{ as }\varepsilon\to 0}\ge\min_{S:\pi(S)>0}\underbrace{\frac{\log P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to c_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}. \tag{A.5}$$

Taking limits of (A.5) as $\varepsilon\to 0$, we obtain

$$\min_{S:\pi(S)>0}c_S(\sigma,\sigma')\ge c(\sigma,\sigma')\ge\min_{S:\pi(S)>0}c_S(\sigma,\sigma'),$$

and therefore $c(\sigma,\sigma')=\min_{S:\pi(S)>0}c_S(\sigma,\sigma')$.
‖

Proof of Lemma 2. Consider $\sigma,\sigma',\tilde{\sigma}\in\Sigma$, such that $V_B(\sigma)\subseteq V_A(\tilde{\sigma})$.
As $c_1(\cdot,\cdot)$ is asymmetric, there exists $\bar{\sigma}\in\Sigma$ such that $V_A(\tilde{\sigma})\subseteq V_A(\bar{\sigma})$, $V_B(\sigma')\subseteq V_A(\bar{\sigma})$ and

$$c_1(\sigma,\sigma')\ge c_1(\tilde{\sigma},\bar{\sigma}). \tag{A.6}$$

As $c_2(\cdot,\cdot)$ is asymmetric, there exists $\bar{\bar{\sigma}}\in\Sigma$ such that $V_A(\tilde{\sigma})\subseteq V_A(\bar{\bar{\sigma}})$, $V_B(\sigma')\subseteq V_A(\bar{\bar{\sigma}})$ and

$$c_2(\sigma,\sigma')\ge c_2(\tilde{\sigma},\bar{\bar{\sigma}}). \tag{A.7}$$

Consequently, we have that

$$\begin{aligned}c(\sigma,\sigma')&\underbrace{=}_{\text{by defn of }c}\min\{c_1(\sigma,\sigma'),c_2(\sigma,\sigma')\}\\&\underbrace{\ge}_{\text{by (A.6) and (A.7)}}\min\{c_1(\tilde{\sigma},\bar{\sigma}),c_2(\tilde{\sigma},\bar{\bar{\sigma}})\}\\&\ge\min\big\{\min\{c_1(\tilde{\sigma},\bar{\sigma}),c_2(\tilde{\sigma},\bar{\sigma})\},\min\{c_1(\tilde{\sigma},\bar{\bar{\sigma}}),c_2(\tilde{\sigma},\bar{\bar{\sigma}})\}\big\}\\&\underbrace{=}_{\text{by defn of }c}\min\big\{c(\tilde{\sigma},\bar{\sigma}),c(\tilde{\sigma},\bar{\bar{\sigma}})\big\},\end{aligned}$$

so $c(\sigma,\sigma')\ge c(\tilde{\sigma},\bar{\sigma})$ or $c(\sigma,\sigma')\ge c(\tilde{\sigma},\bar{\bar{\sigma}})$, and the condition for c to be asymmetric is satisfied. ‖

Proof of Theorem 1. Keep in mind that, as $\varepsilon<1$, $\log\varepsilon<0$, and let $\frac{\log 0}{\log\varepsilon}:=\infty$. Then, for all $\varepsilon>0$,

$$\begin{aligned}
&\min\left\{\underbrace{\frac{\log\lambda}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0}+\underbrace{\frac{\log\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to\tilde{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0},\underbrace{\frac{\log(1-\lambda)}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0}+\underbrace{\frac{\log\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to\bar{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}\right\} \qquad (A.8)\\
&=\min\left\{\frac{\log\left(\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon},\frac{\log\left((1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon}\right\}\\
&=\frac{\log\left(\max\left\{\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma'),(1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right\}\right)}{\log\varepsilon}\\
&\geq\frac{\log\left(\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')+(1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon}\\
&=\underbrace{\frac{\log P^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to c(\sigma,\sigma')\text{ as }\varepsilon\to 0}=\frac{\log\left(\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')+(1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon}\\
&\geq\frac{\log\left(2\max\left\{\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma'),(1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right\}\right)}{\log\varepsilon}\\
&=\min\left\{\frac{\log\left(2\lambda\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon},\frac{\log\left(2(1-\lambda)\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')\right)}{\log\varepsilon}\right\}\\
&=\min\left\{\underbrace{\frac{\log(2\lambda)}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0}+\underbrace{\frac{\log\tilde{P}^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to\tilde{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0},\underbrace{\frac{\log(2(1-\lambda))}{\log\varepsilon}}_{\to 0\text{ as }\varepsilon\to 0}+\underbrace{\frac{\log\bar{P}^{\varepsilon}_{S}(\sigma,\sigma')}{\log\varepsilon}}_{\to\bar{c}_S(\sigma,\sigma')\text{ as }\varepsilon\to 0}\right\}.
\end{aligned}$$

Taking limits of (A.8) as $\varepsilon\to 0$, we obtain

$$\min\{\tilde{c}_S(\sigma,\sigma'),\bar{c}_S(\sigma,\sigma')\}\geq c_S(\sigma,\sigma')\geq\min\{\tilde{c}_S(\sigma,\sigma'),\bar{c}_S(\sigma,\sigma')\},$$

and therefore $c_S(\sigma,\sigma')=\min\{\tilde{c}_S(\sigma,\sigma'),\bar{c}_S(\sigma,\sigma')\}$.

As $\tilde{P}_S$ and $\bar{P}_S$ are asymmetric, $\tilde{c}_S$ and $\bar{c}_S$ are asymmetric.

Lemma 2 then implies that c_S is asymmetric, therefore P_S is asymmetric. ‖

Proof of Theorem 2. By assumption, for all S such that $\pi(S)>0$, P_S is asymmetric, so c_S is asymmetric.

By Lemma 1, $c=\min_{S:\pi(S)>0}c_S$. We shall show that c is asymmetric, therefore P is asymmetric.

Let $\{S:\pi(S)>0\}=\{S_1,S_2,\ldots,S_n\}$ and define cost functions $\hat{c}_1=c_{S_1}$, $\hat{c}_m:=\min\{\hat{c}_{m-1},c_{S_m}\}=\min\{c_{S_1},\ldots,c_{S_m}\}$ for $m=2,\ldots,n$. In particular,

$$\hat{c}_n\underbrace{=}_{\substack{\text{by defn}\\ \text{of }c_n}}\min\{c_{S_1},\ldots,c_{S_n}\}\underbrace{=}_{\substack{\text{by defn}\\ \text{of }\{S_1,\ldots,S_n\}}}\min_{S:\pi(S)>0}c_S\underbrace{=}_{\text{by Lemma 1}}c,$$

We complete the proof by showing, by induction, that $\hat{c}_m$ is asymmetric for $m=2,\ldots,n$. By assumption, $\hat{c}_1=c_{S_1}$ is asymmetric. Assume $\hat{c}_{m-1}$ is asymmetric for some $m\leq n$. Then $\hat{c}_m=\min\{\hat{c}_{m-1},c_{S_m}\}$ is asymmetric by Lemma 2. ‖

Proof of Theorem 3. Consider $\sigma,\sigma',\tilde{\sigma}\in\Sigma$, such that $V_B(\sigma)\subseteq V_A(\tilde{\sigma})$. Let $c_{S\cup T}$ be the cost function of $P_{S\cup T}$.

If $c_{S\cup T}(\sigma,\sigma')=\infty$, we are done as then $c_{S\cup T}(\sigma,\sigma')\geq c_{S\cup T}(\tilde{\sigma},\bar{\sigma})$ for any $\bar{\sigma}$.

If $c_{S\cup T}(\sigma,\sigma')<\infty$, then $\sigma'=(\sigma'_S,\sigma'_T,\sigma_{V\setminus(S\cup T)})$.

As cost functions are defined using logs of transition probabilities, it follows from the definition of $P_{S\cup T}$ that

$$c_{S\cup T}(\sigma,\sigma')=c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S}))+c_T(\sigma,(\sigma'_T,\sigma_{V\setminus T})). \qquad (A.9)$$

As (A.9) is finite, each of its terms are finite, so

$$c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S}))<\infty,\qquad c_T(\sigma,(\sigma'_T,\sigma_{V\setminus T}))<\infty. \qquad (A.10)$$

By asymmetry of c_S, there exists $\bar{\bar{\sigma}}$ such that $V_A(\tilde{\sigma})\subseteq V_A(\bar{\bar{\sigma}})$, $V_B((\sigma'_S,\sigma_{V\setminus S}))\subseteq V_A(\bar{\bar{\sigma}})$, and

$$c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S}))\geq c_S(\tilde{\sigma},\bar{\bar{\sigma}}) \tag{A.11}$$

Inequalities (A.10) and (A.11) imply that $c_S(\tilde{\sigma},\bar{\bar{\sigma}})<\infty$, which implies that $\bar{\bar{\sigma}}=(\bar{\sigma}_S,\tilde{\sigma}_{V\setminus S})$ for some $\bar{\sigma}_S$. Therefore,

$$c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S}))\geq c_S(\tilde{\sigma},(\bar{\sigma}_S,\tilde{\sigma}_{V\setminus S})). \tag{A.12}$$

Similarly, by asymmetry of c_T, we obtain $(\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T})$ such that $V_A(\tilde{\sigma})\subseteq V_A((\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T}))$, $V_B((\sigma'_T,\sigma_{V\setminus T}))\subseteq V_A((\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T}))$, and

$$c_T(\sigma,(\sigma'_T,\sigma_{V\setminus T}))\geq c_T(\tilde{\sigma},(\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T})). \tag{A.13}$$

Let $\bar{\sigma}=(\bar{\sigma}_S,\bar{\sigma}_T,\tilde{\sigma}_{V\setminus(S\cup T)})$.

As $V_B((\sigma'_S,\sigma_{V\setminus S}))\subseteq V_A(\bar{\bar{\sigma}})=V_A((\bar{\sigma}_S,\tilde{\sigma}_{V\setminus S}))$ and $V_B((\sigma'_T,\sigma_{V\setminus T}))\subseteq V_A((\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T}))$, it must be that $V_B(\sigma')=V_B((\sigma'_S,\sigma'_T,\sigma_{V\setminus(S\cup T)}))\subseteq V_A((\bar{\sigma}_S,\bar{\sigma}_T,\tilde{\sigma}_{V\setminus(S\cup T)}))=V_A(\bar{\sigma})$.

Similarly, as $V_A(\tilde{\sigma})\subseteq V_A(\bar{\bar{\sigma}})=V_A((\bar{\sigma}_S,\tilde{\sigma}_{V\setminus S}))$ and $V_A(\tilde{\sigma})\subseteq V_A((\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T}))$, it must be that $V_A(\tilde{\sigma})\subseteq V_A((\bar{\sigma}_S,\bar{\sigma}_T,\tilde{\sigma}_{V\setminus(S\cup T)}))=V_A(\bar{\sigma})$.

Finally,

$$\begin{aligned} c_{S\cup T}(\sigma,\sigma') &= c_S(\sigma,(\sigma'_S,\sigma_{V\setminus S}))+c_T(\sigma,(\sigma'_T,\sigma_{V\setminus T})) \\ \underbrace{\geq}_{\text{by (A.12) and (A.13)}} & \quad c_S(\tilde{\sigma},(\bar{\sigma}_S,\tilde{\sigma}_{V\setminus S}))+c_T(\tilde{\sigma},(\bar{\sigma}_T,\tilde{\sigma}_{V\setminus T}))=c_{S\cup T}(\tilde{\sigma},\bar{\sigma}), \end{aligned}$$

therefore, $c_{S\cup T}$ is asymmetric. ‖

B. ADDITIONAL DEFINITIONS AND RESULTS FOR INDIVIDUAL BEHAVIOURAL RULES

This section considers $S=\{i\}$, that is when only a single player updates his strategy (*e.g.* player i in Figure 3). Recall that, given a strategy profile σ, $\sigma^{(i)}$ denotes the strategy profile which is identical to σ except for the strategy of player i. That is, $\sigma^{(i)}(j)=\sigma(j)$ for all $j\neq i$, and $\sigma^{(i)}(i)\neq\sigma(i)$.

B.1. *Proof that Definitions 2 and 3 are equivalent*

Here, we show that when $S=\{i\}$, the two definitions of asymmetry given in Section 2 are equivalent.

Lemma B.1 *$c_{\{i\}}(\cdot,\cdot)$ is asymmetric towards A under Definition 2 if and only if it is asymmetric towards A under Definition 3.*

Proof of Lemma B.1. We shall consider Definition 3 case by case, showing that in some cases the conditions in the definition are always satisfied and that the remaining case reduces to Definition 2.

Consider $\sigma,\sigma',\tilde{\sigma}\in\Sigma$, such that $V_B(\sigma)\subseteq V_A(\tilde{\sigma})$.

If $c_{\{i\}}(\sigma,\sigma')=\infty$, then letting $\bar{\sigma}=\sigma^A$, we have $V_A(\tilde{\sigma})\subseteq V_A(\bar{\sigma})$, $V_B(\sigma')\subseteq V_A(\bar{\sigma})$, and $c_{\{i\}}(\sigma,\sigma')\geq c_{\{i\}}(\tilde{\sigma},\bar{\sigma})$. The condition in Definition 3 is satisfied.

If $c_{\{i\}}(\sigma,\sigma')<\infty$, then either $\sigma'=\sigma$ or $\sigma'=\sigma^{(i)}$.

If $\sigma'=\sigma$ or $\sigma'=\sigma^{(i)}$ for $i\in V_A(\tilde{\sigma})$, then let $\bar{\sigma}=\tilde{\sigma}$. We have $V_A(\tilde{\sigma})\subseteq V_A(\bar{\sigma})$, $V_B(\sigma')\subseteq V_A(\bar{\sigma})$, and $c_{\{i\}}(\sigma,\sigma')\geq 0=c_{\{i\}}(\tilde{\sigma},\bar{\sigma})$. The condition in Definition 3 is satisfied.

Noting that $i\in V_B(\sigma)$ implies that $i\in V_A(\tilde{\sigma})$ (so the preceding case would apply), we have one remaining case, $\sigma'=\sigma^{(i)}$ for $i\in V_A(\sigma)$, $i\in V_B(\tilde{\sigma})$. This implies that $i\in V_B(\sigma')$, so if we are to have $V_B(\sigma')\subseteq V_A(\bar{\sigma})$, it must be the case that $i\in V_A(\bar{\sigma})$. However, the only $\bar{\sigma}$ that could possibly satisfy both this condition and $c_{\{i\}}(\tilde{\sigma},\bar{\sigma})<\infty$ is $\bar{\sigma}=\tilde{\sigma}^{(i)}$. Therefore, the condition in Definition 3 is satisfied if and only if $c_{\{i\}}(\sigma,\sigma^{(i)})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$, which is exactly the condition in Definition 2. ‖

B.2. *Considering asymmetry as an implication of weak asymmetry and supermodularity*

When $S=\{i\}$, it will help to consider asymmetry as an implication of two other properties: weak asymmetry and supermodularity.

Definition B.1 *$c_{\{i\}}(\cdot,\cdot)$ is* weakly asymmetric *towards A if, for all $\sigma,\hat{\sigma}\in\Sigma$ such that $V_B(\sigma)=V_A(\hat{\sigma})$, if $i\in V_A(\sigma)$, then $c_{\{i\}}(\sigma,\sigma^{(i)})\geq c_{\{i\}}(\hat{\sigma},\hat{\sigma}^{(i)})$.*

States σ and $\hat{\sigma}$ in Definition B.1 mirror each other in that players who play A at σ, play B at $\hat{\sigma}$, and players who play B at σ, play A at $\hat{\sigma}$ (see Figure 3[i,ii]). Weak asymmetry means that a switch from A to B by player i from state σ is weakly less probable than a switch from B to A by player i from state $\hat{\sigma}$.

Definition B.2 *$c_{\{i\}}(\cdot,\cdot)$ is* supermodular *towards A if, for all $\hat{\sigma},\tilde{\sigma}\in\Sigma$ such that $V_A(\hat{\sigma})\subseteq V_A(\tilde{\sigma})$, if $i\in V_B(\tilde{\sigma})$, then $c_{\{i\}}(\hat{\sigma},\hat{\sigma}^{(i)})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$.*

States $\hat{\sigma}$, $\tilde{\sigma}$ in Definition B.2 are such that all players who play A at $\hat{\sigma}$ also play A at $\tilde{\sigma}$ (see Figure 3[ii,iii]). Let player i be any player who plays B at both states. Supermodularity means that a switch from B to A by player i from state $\hat{\sigma}$ is weakly less probable than a switch from B to A by player i from state $\tilde{\sigma}$. That is, switches by player i from B to A are weakly more probable when more of the other players are playing A.

Lemma B.2 *If $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric towards A and supermodular towards A, then it is asymmetric towards A.*

The notation chosen for Definitions B.1 and B.2 has been chosen to facilitate understanding of Lemma B.2 in terms of these definitions. Specifically, if we consider σ, $\hat{\sigma}$, $\tilde{\sigma}$ as given in Definitions B.1 and B.2, we have

$$c_{\{i\}}(\sigma,\sigma^{(i)}) \underbrace{\geq}_{\text{weak asymmetry}} c_{\{i\}}(\hat{\sigma},\hat{\sigma}^{(i)}) \underbrace{\geq}_{\text{supermodularity}} c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)}), \tag{B.1}$$

which implies the condition $c_{\{i\}}(\sigma,\sigma^{(i)})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$ for asymmetry given in Definition 2. As it is possible that $\tilde{\sigma}=\hat{\sigma}$, asymmetry implies weak asymmetry. In contrast, (B.1) tells us that if weak asymmetry holds strictly, then supermodularity can be violated by some amount while retaining asymmetry.

Proof of Lemma B.2. Consider σ, σ', $\tilde{\sigma}$ such that $V_B(\sigma)\subseteq V_A(\tilde{\sigma})$, $i\in V_A(\sigma)$, $i\in V_B(\tilde{\sigma})$.

Let $\hat{\sigma}$ be such that $V_B(\sigma)=V_A(\hat{\sigma})$. Then, as $c_{\{i\}}$ is weakly asymmetric, by Definition B.1, we have that $c_{\{i\}}(\sigma,\sigma^{(i)})\geq c_{\{i\}}(\hat{\sigma},\hat{\sigma}^{(i)})$.

Note that $V_A(\hat{\sigma})\subseteq V_A(\tilde{\sigma})$. Then, as $c_{\{i\}}$ is supermodular, by Definition B.2, we have that $c_{\{i\}}(\hat{\sigma},\hat{\sigma}^{(i)})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$.

Combining the inequalities above, we have $c_{\{i\}}(\sigma,\sigma^{(i)})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})$, satisfying the condition for asymmetry of Definition 2. ‖

B.3. *Weak asymmetry of every agent's rule does not imply stochastic stability*

Here, we show that every agent having a behavioural rule that is weakly asymmetric towards A does not imply stochastic stability of σ^A.

Let there be two players, $|V|=2$. For each player $i,j\neq i$, let $c_{\{i\}}(\sigma,\sigma^{(i)})=a(\sigma)+d(\sigma)$, where $a(\sigma)=1$ if $\sigma_i=A$ and is zero otherwise, and $d(\sigma)=2$ if $\sigma_i\neq\sigma_j$ and is zero otherwise. There are two aspects to this cost function. First, $a(\cdot)$ gives a bias towards A. Second, $d(\cdot)$ gives a bias towards playing a strategy different to that of the other player.

Letting $\sigma=\sigma^A$ and $\tilde{\sigma}=(\tilde{\sigma}_i,\tilde{\sigma}_j)=(B,A)$, we see that $c_{\{i\}}(\sigma,\sigma^{(i)})=1$ and $c_{\{i\}}(\tilde{\sigma},\tilde{\sigma}^{(i)})=2$, so Definition 2 is not satisfied and $c_{\{i\}}$ is not asymmetric towards A.

However, it can be checked that Definition B.1 is satisfied, so $c_{\{i\}}$ is weakly asymmetric towards A.

Note that a transition from any profile at which both players play the same strategy to a profile at which the players play different strategies has a cost of 0 or 1.

However, a transition from any profile at which the players play different strategies to a profile at which both players play the same strategy has a cost of 2 or 3.

Hence, for small values of ε, the invariant measure of the process concentrates on the set of strategy profiles at which the players play different strategies. That is, σ^A is not stochastically stable.

C. PROOFS FOR PAYOFF-DIFFERENCE BASED CHOICE

C.1. *Explicit choice probabilities*

For completeness, we here give some explicit choice probabilities, that is specific $P^{\varepsilon}_{\{i\}}$ that satisfy the definition of payoff-difference based rules given in Section 4.

For non-decreasing $\Upsilon_i(\cdot):\mathbb{R}^V \to \mathbb{R}_+$ and constant (with respect to ε) $d_i^\sigma \in (0,1)$, $\sigma \in \Sigma$, let

$$P^{\varepsilon}_{\{i\}}(\sigma,\sigma) = 1 - d_i^\sigma \varepsilon^{\Upsilon_i(D_i^\sigma)} \quad \text{and} \quad P^{\varepsilon}_{\{i\}}(\sigma,\sigma^{(i)}) = d_i^\sigma \varepsilon^{\Upsilon_i(D_i^\sigma)}, \tag{C.1}$$

with the convention that $0^0=1$ so that $P^{\varepsilon}_{\{i\}}$ is continuous in ε at $\varepsilon=0$. Such rules satisfy the restriction on behaviour that if a transition is at least as good (measured by changes in payoff) for everybody as another transition, then the first transition should be no less likely to occur than the second.

A strictly positive $\Upsilon_i(D_i^\sigma)$ implies that the probability of a transition from σ to $\sigma^{(i)}$ approaches zero as ε approaches zero. In contrast, $\Upsilon_i(D_i^\sigma)=0$ implies that the probability of a transition from σ to $\sigma^{(i)}$ is strictly positive even under the unperturbed process $P^0_{\{i\}}$. Substituting (C.1) into the definition of a cost function, we obtain (4.1), the definition of payoff-difference based rules in Section 4.

C.2. *Proof of Proposition 1*

The proof of Proposition 1 proceeds by considering weak asymmetry and supermodularity as defined in Appendix B. These two properties then imply asymmetry by Lemma B.2.

Lemma C.1 *If player i follows a payoff-difference based rule, A is RD$_i$ and*

(i) Υ_i is self-regarding, or
(ii) A is ARD$_{ij}$ for all j,

then $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric towards A.

Proof of Lemma C.1. Let σ, $\hat\sigma$ be such that $V_A(\sigma)=V_B(\hat\sigma)$, $\sigma(i)=A$.
Consider the elements of D_i^σ,

$$\begin{aligned}(D_i^\sigma)_j &= U_j(\sigma) - U_j(\sigma^{(i)}) \\ &= \begin{cases} u_{ji}(A,A)-u_{ji}(A,B) & \text{if } j\neq i, \sigma(j)=A, \\ -\big(u_{ji}(B,B)-u_{ji}(B,A)\big) & \text{if } j\neq i, \sigma(j)=B, \\ \sum_{k\in V_A(\sigma)\setminus\{i\}}\Big(u_{ik}(A,A)-u_{ik}(B,A)\Big) & \\ -\sum_{k\in V_B(\sigma)\setminus\{i\}}\Big(u_{ik}(B,B)-u_{ik}(A,B)\Big) & \text{if } j=i, \end{cases}\end{aligned} \tag{C.2}$$

and the elements of $D_i^{\hat\sigma}$,

$$\begin{aligned}(D_i^{\hat\sigma})_j &= U_j(\hat\sigma) - U_j(\hat\sigma^{(i)}) \\ &= \begin{cases} -\big(u_{ji}(A,A)-u_{ji}(A,B)\big) & \text{if } j\neq i, \hat\sigma(j)=A, \\ u_{ji}(B,B)-u_{ji}(B,A) & \text{if } j\neq i, \hat\sigma(j)=B, \\ -\sum_{k\in V_A(\hat\sigma)\setminus\{i\}}\Big(u_{ik}(A,A)-u_{ik}(B,A)\Big) & \\ +\sum_{k\in V_B(\hat\sigma)\setminus\{i\}}\Big(u_{ik}(B,B)-u_{ik}(A,B)\Big) & \text{if } j=i. \end{cases}\end{aligned} \tag{C.3}$$

Noting that $V_A(\hat\sigma)=V_B(\sigma)$ and $V_B(\hat\sigma)=V_A(\sigma)$, we can subtract (C.3) from (C.2) to get

$$\begin{aligned}&(D_i^\sigma - D_i^{\hat\sigma})_j \\ &= \begin{cases} \big(u_{ji}(A,A)-u_{ji}(A,B)\big)-\big(u_{ji}(B,B)-u_{ji}(B,A)\big) & \text{if } j\neq i, \sigma(j)=A, \\ \big(u_{ji}(A,A)-u_{ji}(A,B)\big)-\big(u_{ji}(B,B)-u_{ji}(B,A)\big) & \text{if } j\neq i, \sigma(j)=B, \\ \sum_{k\in V\setminus\{i\}}\Big(\Big(u_{ik}(A,A)-u_{ik}(B,A)\Big)-\Big(u_{ik}(B,B)-u_{ik}(A,B)\Big)\Big) & \text{if } j=i. \end{cases}\end{aligned} \tag{C.4}$$

If A is RD$_i$, then, from the third case of (C.4), we have that $(D_i^\sigma - D_i^{\hat\sigma})_i \geq 0$, so $(D_i^\sigma)_i \geq (D_i^{\hat\sigma})_i$.

If Υ_i is self-regarding, then $(D_i^\sigma)_i \geq (D_i^{\hat\sigma})_i$ implies that $\Upsilon_i(D_i^\sigma) \geq \Upsilon_i(D_i^{\hat\sigma})$ and therefore, by (4.1), $c_{\{i\}}(\sigma,\sigma^{(i)}) \geq c_{\{i\}}(\hat\sigma,\hat\sigma)$. That is, $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric, proving Lemma C.1[i].

If A is ARD$_{ij}$ for all j, then, from the first and second cases of (C.4), we have that $(D_i^\sigma - D_i^{\hat\sigma})_j \geq 0$ and $(D_i^\sigma)_j \geq (D_i^{\hat\sigma})_j$ for all $j\neq i$. Therefore $D_i^\sigma \geq D_i^{\hat\sigma}$, and as Υ_i is non-decreasing, $\Upsilon_i(D_i^\sigma) \geq \Upsilon_i(D_i^{\hat\sigma})$ and therefore, by (4.1), $c_{\{i\}}(\sigma,\sigma^{(i)}) \geq c_{\{i\}}(\hat\sigma,\hat\sigma)$. That is, $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric, proving Lemma C.1[ii]. ‖

Lemma C.2 *If player i follows a payoff-difference based rule, then $c_{\{i\}}(\cdot,\cdot)$ is supermodular towards A.*

Proof of Lemma C.2. Let $\hat{\sigma}$, $\tilde{\sigma}$ be such that $\hat{\sigma}(i)=\tilde{\sigma}(i)=B$, $V_A(\hat{\sigma})\subseteq V_A(\tilde{\sigma})$.
Using (C.3) for both $D_i^{\hat{\sigma}}$ and $D_i^{\tilde{\sigma}}$ gives

$$(D_i^{\hat{\sigma}}-D_i^{\tilde{\sigma}})_j \tag{C.5}$$
$$=\begin{cases}\big(u_{ji}(A,A)-u_{ji}(A,B)\big)-\big(u_{ji}(A,A)-u_{ji}(A,B)\big)=0 & \text{if } j\neq i, \hat{\sigma}(j)=A,\\ \big(u_{ji}(B,B)-u_{ji}(B,A)\big)-\big(u_{ji}(B,B)-u_{ji}(B,A)\big)=0 & \text{if } j\neq i, \tilde{\sigma}(j)=B,\\ \big(u_{ji}(A,A)-u_{ji}(A,B)\big)+\big(u_{ji}(B,B)-u_{ji}(B,A)\big) & \text{if } j\neq i, \tilde{\sigma}(j)=A, \hat{\sigma}(j)=B,\\ \sum_{k\in V_A(\tilde{\sigma})\setminus\{i\}}\Big(u_{ik}(A,A)-u_{ik}(B,A)\Big) & \\ -\sum_{k\in V_A(\hat{\sigma})\setminus\{i\}}\Big(u_{ik}(A,A)-u_{ik}(B,A)\Big) & \\ +\sum_{k\in V_B(\hat{\sigma})\setminus\{i\}}\Big(u_{ik}(B,B)-u_{ik}(A,B)\Big) & \\ -\sum_{k\in V_B(\tilde{\sigma})\setminus\{i\}}(\tilde{\sigma})\Big(u_{ik}(B,B)-u_{ik}(A,B)\Big) & \text{if } j=i.\end{cases}$$

The third case of (C.5) is nonnegative by (2.2). The sum of the first two lines of the fourth case is non-negative by $V_A(\hat{\sigma})\subseteq V_A(\tilde{\sigma})$ and (2.2). The sum of the final two lines of the fourth case is nonnegative by a similar argument. So every element $(D_i^{\hat{\sigma}}-D_i^{\tilde{\sigma}})_j$ is non-negative, $D_i^{\hat{\sigma}}\geq D_i^{\tilde{\sigma}}$. As Υ_i is non-decreasing, $\Upsilon_i(D_i^{\hat{\sigma}})\geq\Upsilon_i(D_i^{\tilde{\sigma}})$ and therefore, by (4.1), $c_{\{i\}}(\hat{\sigma},\hat{\sigma})\geq c_{\{i\}}(\tilde{\sigma},\tilde{\sigma})$. That is, $c_{\{i\}}(\cdot,\cdot)$ is supermodular. ‖

Proof of Proposition 1. By Lemmas C.1 and C.2, $c_{\{i\}}(\cdot,\cdot)$ is weakly asymmetric and supermodular, so by Lemma B.2, $c_{\{i\}}(\cdot,\cdot)$ is asymmetric. ‖

Proof of Corollary 1. As A is RD_i for all $i\in V$ and all $i\in V$ follow self-regarding payoff-difference based rules, Proposition 1[i] implies that $c_{\{i\}}$ is asymmetric for all $i\in V$. As, by assumption, $\pi(S)>0$ if and only if $S=\{i\}$ for $i\in V$, Theorem 2 then implies that $c=\min_{S:\pi(S)>0}c_S$ is asymmetric. By Theorem P, σ^A is stochastically stable. ‖

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

REFERENCES

AGASTYA, M. (1997), “Adaptive Play in Multiplayer Bargaining Situations”, *Review of Economic Studies*, **64**, 411–426.

AGASTYA, M. (1999), “Perturbed Adaptive Dynamics in Coalition Form Games”, *Journal of Economic Theory*, **89**, 207–233.

ALGER, I. and WEIBULL, J. W. (2013), “Homo Moralis—Preference Evolution under Incomplete Information and Assortative Matching”, *Econometrica*, **81**, 2269–2302.

ALGER, I. and WEIBULL, J. W. (2016), “Evolution and Kantian Morality”, *Games and Economic Behavior*, **98**, 56–67.

ALOS-FERRER, C. and ANIA, A. B. (2005), “The Evolutionary Stability of Perfectly Competitive Behavior”, *Economic Theory*, **26**, 497–516.

ALÓS-FERRER, C. and NETZER, N. (2010), “The Logit-Response Dynamics”, *Games and Economic Behavior*, **68**, 413–427.

ALÓS-FERRER, C. and NETZER, N. (2015), “Robust Stochastic Stability” *Economic Theory*, **58**, 31–57.

ALÓS-FERRER, C. and SCHLAG, K. (2009), “Imitation and Learning”, in Anand, P., Pattanaik, P., and Puppe, C., (eds) *Handbook of Rational and Social Choice* (Oxford, UK: Oxford University Press).

ALÓS-FERRER, C. and WEIDENHOLZER, S. (2008), “Contagion and Efficiency”, *Journal of Economic Theory*, **143**, 251–274.

ALÓS-FERRER, C. and WEIDENHOLZER, S. (2014), "Imitation and the Role of Information in Overcoming Coordination Failures", *Games and Economic Behavior*, **87**, 397–411.

ARIELI, I. and YOUNG, H. P. (2016), "Stochastic Learning Dynamics and Speed of Convergence in Population Games", *Econometrica*, **84**, 627–676.

AXELROD, R. (1984), *The Evolution of Cooperation* (New York, NY, USA: Basic Books, revised 2006 edition).

BERGSTROM, T. C. (1995), "On the Evolution of Altruistic Ethical Rules for Siblings", *American Economic Review*, **85**, 58–81.

BILANCINI, E. and BONCINELLI, L. (2020), "The Evolution of Conventions under Condition-dependent Mistakes", *Economic Theory*, **69**, 497–521.

BLUME, L. E. (1993), "The Statistical Mechanics of Strategic Interaction", *Games and Economic Behavior*, **5**, 387–424.

BLUME, L. E. (1996), "Population Games" (Working Papers 96-04-022, Santa Fe Institute).

BLUME, L. E. (2003), "How Noise Matters", *Games and Economic Behavior*, **44**, 251–271.

CASON, T. N., FRIEDMAN, D., and HOPKINS, E. (2013), "Cycles and Instability in a Rock–Paper–Scissors Population Game: A Continuous Time Experiment", *Review of Economic Studies*, **81**, 112–136.

COURNOT, A.-A. (1838), *Recherches sur les principes mathématiques de la théorie des richesses par Augustin Cournot* (Chez L. Hachette).

DOKUMACI, E. and SANDHOLM, W. H. (2011), "Large Deviations and Multinomial Probit Choice", *Journal of Economic Theory*, **146**, 2151–2158.

ELLISON, G. (1993), "Learning, Local Interaction, and Coordination", *Econometrica*, **61**, 1047–1071.

ELLISON, G. (2000), "Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution", *Review of Economic Studies*, **67**, 17–45.

ELLISON, G. and FUDENBERG, D. (1995), Word-of-Mouth Communication and Social Learning. *The Quarterly Journal of Economics*, **110**, 93–125.

ELY, J. C. and SANDHOLM, W. H. (2005), "Evolution in Bayesian Games I: Theory", *Games and Economic Behavior*, **53**, 83–109.

FOSTER, D. and YOUNG, H. P. (1990), "Stochastic Evolutionary Game Dynamics", *Theoretical Population Biology*, **38**, 219–232.

FREIDLIN, M. I. and WENTZELL, A. D. (1984), *Random Perturbations of Dynamical Systems,* 2nd ed. (1998). (Berlin, Germany: Springer).

FRIEDMAN, D., HUCK, S., OPREA, R., et al. (2015), "From Imitation to Collusion: Long-Run Learning in a Low-Information Environment", *Journal of Economic Theory*, **155**, 185–205.

GILBERT, M. (1981), "Game Theory and Convention", *Synthese*, **46**, 41–93.

HARSANYI, J. C. and SELTEN, R. (1988), *A General Theory of Equilibrium Selection in Games*, Vol. 1 (Cambridge, MA: MIT Press Books).

HWANG, S.-H., LIM, W., NEARY, P., et al. (2018). 'Conventional Contracts, Intentional Behavior and Logit Choice: Equality without Symmetry", *Games and Economic Behavior*, **110**, 273–294.

KANDORI, M., MAILATH, G. J., and ROB, R. (1993), "Learning, Mutation, and Long Run Equilibria in Games", *Econometrica*, **61**, 29–56.

KHAN, A. (2014), "Coordination under Global Random Interaction and Local Imitation", *International Journal of Game Theory*, **43**, 721–745.

KREINDLER, G. E. and YOUNG, H. P. (2013), "Fast Convergence in Evolutionary Equilibrium Selection" *Games and Economic Behavior*, **80**, 39–67.

LEHMANN, L., KELLER, L., and SUMPTER, D. J. T. (2007), "The Evolution of Helping and Harming on Graphs: The Return of the Inclusive Fitness Effect", *Journal of Evolutionary Biology*, **20**, 2284–2295.

LEWIS, D. (1969), *Convention: A Philosophical Study* (Cambridge, MA, USA: Harvard University Press).

LIM, W. and NEARY, P. R. (2016), "An Experimental Investigation of Stochastic Adjustment Dynamics", *Games and Economic Behavior*, **100**, 208–219.

MALAWSKI, M. (1989), *Some Learning Processes in Population Games* (PhD thesis, University of Bonn, Germany).

MARUTA, T. (2002), "Binary Games with State Dependent Stochastic Choice", *Journal of Economic Theory*, **103**, 351–376.

MÄS, M. and NAX, H. H. (2016), "A Behavioral Study of "Noise" in Coordination Games", *Journal of Economic Theory*, **162**, 195 – 208.

MATROS, A. (2003), "Clever Agents in Adaptive Learning", *Journal of Economic Theory*, **111**, 110–124.

NASH, J. (1951), "Non-cooperative Games", *Annals of Mathematics*, **54**, 286–295.

NASH, J. F. (1950), "Equilibrium Points in n-Person Games", *Proceedings of the National Academy of Sciences*, **36**, 48–49.

NAX, H. H., BURTON-CHELLEW, M. N., WEST, S. A., et al. (2016), "Learning in a Black Box", *Journal of Economic Behavior & Organization*, **127**, 1–15.

NAX, H. H. and NEWTON, J. (2019), "Risk Attitudes and Risk Dominance in the Long Run", *Games and Economic Behavior*, **116**, 179–184.

NAX, H. H. and RIGOS, A. (2016), "Assortativity Evolving from Social Dilemmas", *Journal of Theoretical Biology*, **395**, 194–203.

NEWTON, J. (2012a), "Coalitional Stochastic Stability", *Games and Economic Behavior*, **75**, 842–854.

NEWTON, J. (2012b), "Recontracting and Stochastic Stability in Cooperative Games", *Journal of Economic Theory*, **147**, 364–381.

NEWTON, J. (2017), “The Preferences of Homo Moralis are Unstable under Evolving Assortativity”, *International Journal of Game Theory*, **46**, 583–589.

NEWTON, J. (2018), “Evolutionary Game Theory: A Renaissance” *Games*, **9**, 31.

NEWTON, J. (2019), “Combining Strictly Asymmetric Choice Rules” (SSRN Working Paper Series 3433736).

NEWTON, J. and ANGUS, S. D. (2015), “Coalitions, Tipping Points and the Speed of Evolution”, *Journal of Economic Theory*, **157**, 172–187.

NEWTON, J. and SAWA, R. (2015), “A One-Shot Deviation Principle for Stability in Matching Problems”, *Journal of Economic Theory*, **157**, 1–27.

NORMAN, T. (2009a), “Skew-symmetry under Simultaneous Revisions”, *International Game Theory Review*, **11**, 471–478.

NORMAN, T. W. (2009b), “Rapid Evolution under Inertia”, *Games and Economic Behavior*, **66**, 865–879.

OHTSUKI, H., HAUERT, C., LIEBERMAN, et al. (2006), “A Simple Rule for the Evolution of Cooperation on Graphs and Social Networks”, *Nature*, **441**, 502–505.

PESKI, M. (2010), “Generalized Risk-Dominance and Asymmetric Dynamics”, *Journal of Economic Theory*, **145**, 216 – 248.

ROBSON, A. J. and VEGA-REDONDO, F. (1996), “Efficient Equilibrium Selection in Evolutionary Games with Random Matching”, *Journal of Economic Theory*, **70**, 65–92.

SÁEZ-MARTI, M. and WEIBULL, J. W. (1999), “Clever Agents in Young’s Evolutionary Bargaining Model”, *Journal of Economic Theory*, **86**, 268–279.

SANDHOLM, W. H. (2010), *Population Games and Evolutionary Dynamics. Economic Learning and Social Evolution)* (Cambridge, MA: MIT Press).

SAWA, R. (2014), “Coalitional Stochastic Stability in Games, Networks and Markets”, *Games and Economic Behavior*, **88**, 90–111.

SAWA, R. and WU, J. (2018), “Prospect Dynamics and Loss Dominance”, *Games and Economic Behavior*, **112**, 98–124.

SCHIPPER, B. C. (2009), “Imitators and Optimizers in Cournot Oligopoly”, *Journal of Economic Dynamics and Control*, **33**, 1981–1990.

SCHLAG, K. H. (1998), “Why Imitate, and If So, How?: A Boundedly Rational Approach to Multi-armed Bandits”, *Journal of Economic Theory*, **78**, 130–156.

SELTEN, R. and APESTEGUIA, J. (2005), “Experimentally Observed Imitation and Cooperation in Price Competition on the Circle”, *Games and Economic Behavior*, **51**, 171–192.

WU, J. (2017), “Political Institutions and the Evolution of Character Traits”, *Games and Economic Behavior*, **106**, 260–276.

YOUNG, H. P. (1993a), “The Evolution of Conventions”, *Econometrica*, **61**, 57–84.

YOUNG, H. P. (1993b), “An Evolutionary Model of Bargaining”, *Journal of Economic Theory*, **59**, 145–168.

YOUNG, H. P. (1998), *Individual Strategy and Social Structure* (Princeton, NJ, USA: Princeton University Press).

YOUNG, H. P. (2011), “The Dynamics of Social Innovation”, *Proceedings of the National Academy of Sciences United States of America*, **108**, 21285–21291.

YOUNG, H. P. and BURKE, M. A. (2001), “Competition and Custom in Economic Contracts: A Case Study of Illinois Agriculture”, *American Economic Review*, **91**, 559–573.

ZUSAI, D. (2018), “Evolutionary Dynamics in Heterogeneous Populations: A General Framework for an Arbitrary Type Distribution”, https://arxiv.org/abs/1805.04897.