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School choice under partial fairness. (English summary)

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The paper considers a variant of the classic discrete college-student matching problem. Each student can be matched to at most one college (referred to as 'schools' in the paper). Each college has a maximum number of students to whom it can be matched. Each student has a preference ranking over colleges. Each college has a priority ranking over students.

A matching is *individually rational* if no student would rather be unmatched than at the college to which they are matched. A matching is *non-wasteful* if there does not exist a student i and college c' such that (i) c' has strictly fewer students than its maximum, and (ii) i would prefer to be matched to c' than to his current college. A matching is *fair* if there do not exist students i, i', and college c', such that (i) student i' is matched to college c', (ii) student i would rather be matched to c' than to his current college, and (iii) college c' gives higher priority to i than it gives to i'. A matching is *stable* if it is individually rational, non-wasteful and fair.

Ignoring the constraints of fairness for some triplets (i, i', c'), the authors attain a concept of *partial fairness*. Given a set of such triplets to ignore, a matching is *partially stable* if it is individually rational, non-wasteful and partially fair.

The paper describes a class of algorithms, *student exchange under partial fairness* (SEPF). Such an algorithm is initiated at a partially stable matching and at each step moves from one partially stable matching to another partially stable matching.

Let μ be a given matching.

Let $\{i_1i_2, i_2i_3, \dots, i_ni_{n+1}\}$ be a set of ordered pairs of students such that $i_1 = i_{n+1}$ and each i_k is matched to college c_k at μ. Define $\{i_1i_2, i_2i_3, \dots, i_ni_{n+1}\}$ to be a *cycle* if each i_k prefers c_{k+1} to c_k, and there is no student $i \neq i_k$ such that (i) i also prefers c_{k+1} to his college at μ, (ii) i is ranked higher than i_k by c_{k+1}, and (iii) (i, i_k, c_{k+1}) is not excluded from the triplets defining partial fairness.

At each step, an SEPF algorithm finds a cycle if one exists, or terminates if it cannot find a cycle. After identifying a cycle, the algorithm moves students i_k to colleges c_{k+1}, then moves to the next iteration of the algorithm.

The definition of a cycle ensures that, at each iteration of the algorithm, we move from one partially stable matching to another partially stable matching. Moreover, all students weakly prefer the output matching to the input.

The paper imposes a monotonicity condition on the triplets that are ignored by partial fairness. Specifically, if college c prefers student i to j' to j and the triplet (i, j, c) is ignored, then (i, j', c) is also ignored.

The authors show that under this monotonicity assumption, the algorithm will not terminate until it reaches a matching that is efficient (from the point of view of the students) within the class of partially stable matchings. Moreover, given an initial matching μ and an efficient partially stable matching μ' that is weakly better than μ for all the students, then an SEPF algorithm exists that outputs μ' given input μ.

This is the main theorem of the paper (Theorem 1). *Jonathan Newton*