Games and Economic Behavior 116 (2019) 179–184

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Risk attitudes and risk dominance in the long run

Heinrich H. Nax [a,b,*], Jonathan Newton [c,**]

[a] Behavioral Game Theory, ETH Zurich, 8092 Zurich, Switzerland
[b] Department of Management, Ca'Foscari University, 30121 Cannaregio, Venice, Italy
[c] Institute of Economic Research, Kyoto University, Sakyo-ku, Kyoto 606-8501, Japan

ARTICLE INFO

Article history:
Received 9 August 2018
Available online 9 May 2019

JEL classification:
C73
D81
D90

Keywords:
Evolution
Conventions
Risk attitudes
Loss aversion
Concave utility
State dependence

ABSTRACT

This paper investigates the role that risk attitudes play in the evolution of conventions in the long run. Risk aversion is shown to be associated with the evolution of maximin conventions, and risk seeking with the evolution of payoff dominant conventions.

1. Introduction

An important item on the agenda of *stochastic evolutionary game theory*[1] since the pioneering works of Foster and Young (1990); Kandori et al. (1993); Young (1993) has been to study dynamics based on strategy updating rules with a firm grounding in human behavior, thus providing a constructive reply to the Bergin and Lipman (1996) critique that different behavioral rules may lead to different outcomes.[2] A recent innovation in this literature comes from Sawa and Wu (2018a), who explore the consequences of individuals being averse to experiences of losses, a well-documented behavioral regularity in decision theory dating back to Kahneman and Tversky (1984). Sawa and Wu find a relationship between (i) loss aversion in a perturbed best response dynamic at the individual level, and (ii) the long run stability of outcomes in strategies that are both risk dominant and maximin.

The current paper takes a simpler approach that turns out to be more general, considering the effect of different risk attitudes on risk dominance and long run behavior in two-strategy games. Specifically, we show that if a strategy is risk dominant and maximin (i.e. what Sawa and Wu refer to as 'loss dominant'), then the risk dominance of that strategy is

* Corresponding author at: Behavioral Game Theory, ETH Zurich, 8092 Zurich, Switzerland.
** Corresponding author.
E-mail addresses: hnax@ethz.ch (H.H. Nax), newton@kier.kyoto-u.ac.jp (J. Newton).

[1] See Newton (2018) for a survey of recent work in evolutionary game theory.

[2] According to Bergin and Lipman (1996), their "result highlights the importance of developing models or other criteria to determine 'reasonable' mutation processes" (p. 944), which is also addressed by an emerging experimental literature (Mäs and Nax, 2016; Lim and Neary, 2016; Hwang et al., 2018).

https://doi.org/10.1016/j.geb.2019.05.001

preserved when payoffs are subjected to concave utility transformations (i.e. under *risk aversion*). This is true even when utility is state-independent, in which case there is no reference-dependent choice and no loss aversion. Consequently, the relationship between loss dominance and loss aversion discovered by Sawa and Wu (2018a) holds because the loss averse utilities which they consider are concave at every state. That is, concavity is what drives these results.

An analogous set of results can be proven for convex utility transformations (i.e. under *risk seeking*). Specifically, if a strategy is risk dominant and payoff dominant, then the risk dominance of that strategy is preserved when payoffs are subjected to convex utility transformations. Each of these results is bidirectional in the sense that, if a strategy is not risk dominant and maximin (respectively, payoff dominant), then there exists a concave (respectively, convex) utility transformation such that the strategy in question is not risk dominant after the transformation.

The next step is to consider state-dependent utility functions. We show that if a strategy is risk dominant at every state, then the convention at which every player plays that strategy is stochastically stable (Foster and Young, 1990) under the best response dynamic with uniform errors, even when utilities are state-dependent. The reason for this robustness is that choice probabilities depend on the sign but not the magnitude of payoff differences, and, in a two-strategy population game, this implies that the basin of attraction of the risk dominant convention remains larger than the basin of attraction of the alternative.

Insofar as the current paper studies the consequences of risk attitudes for stochastic stability, it can be interpreted as an evolutionary analogue of Weinstein (2016), who explores the consequences of risk attitudes for games' rationalizable sets, Nash equilibria and correlated equilibria. In particular, we give conditions for the robustness of stochastic stability to risk attitudes when utility functions are not perfectly known. For example, if all that is known about players' utility is that it is concave, then, whenever risk dominance is a condition for stochastic stability under linear utility, we must also add a maximin condition.[3] For choice rules such as the best response dynamic with uniform errors, this additional condition will suffice to guarantee stochastic stability.

There exists other work that has examined two-strategy coordination games and identified relationships between stochastic stability under perturbed best response dynamics and qualities other than risk dominance.[4] Perhaps most closely related, Maruta (2002) considers best response rules with elements of imitation, under which the resulting conditions for stochastic stability incorporate maximin and payoff dominance as well as risk dominance. A similar result is obtained by Bilancini and Boncinelli (2019), who consider best response with 'condition dependence', that is, under switching rates that decrease in realized payoffs.[5]

2. Model

Let $N = \{1, \ldots, n\}$ be a set of players. Assume that n is even.[6] Every player has strategy set $S = \{A, B\}$. The payoff from playing $s \in S$ against $s' \in S$ equals $a_{ss'} \in \mathbb{R}$. It is assumed that $a_{AA} > a_{BA}$ and $a_{BB} > a_{AB}$. These payoffs are illustrated in Fig. 1[i]. A state $x \in X := \{0, \frac{1}{n}, \ldots, 1\}$ represents the share of players who play strategy B. At state x, payoffs are transformed by a strictly increasing utility function $u^x(\cdot)$, as illustrated in Fig. 1[ii]. The expected utility $U^x(s)$ from playing strategy s against a randomly chosen opponent at state x is thus given by

$$U^x(s) = (1 - x)\, u^x(a_{sA}) + x\, u^x(a_{sB}). \tag{2.1}$$

To keep exposition as clean as possible, assume $U^x(A) \neq U^x(B)$ for any $x \in X$, which holds generically.

Strategies are updated according to a perturbed best response rule parameterized by a small *deviation* probability $\varepsilon \in (0, 1)$. Every period, a single player is selected at random. With probability $(1 - \varepsilon)$, this player best responds, choosing the strategy that maximizes $U^x(\cdot)$. However, with probability ε, he deviates and randomizes uniformly over both strategies. Formally, the process evolves according to the Markov process on X with transition kernel P_ε given by

$$\begin{aligned} P_\varepsilon\left(x,\, x + \frac{1}{n}\right) &= (1 - x)\left((1 - \varepsilon)\, \mathbb{1}_{U^x(A) < U^x(B)} + \frac{\varepsilon}{2}\right) \\ P_\varepsilon\left(x,\, x - \frac{1}{n}\right) &= x\left((1 - \varepsilon)\, \mathbb{1}_{U^x(A) > U^x(B)} + \frac{\varepsilon}{2}\right) \end{aligned} \tag{2.2}$$

[3] A large class of dynamic processes that relates to risk dominance is the class of 'skew-symmetric' rules considered by Blume (2003) and Norman (2009a). Under such a rule, a player's probability of playing a non-best response is weakly decreasing in the expected payoff loss relative to playing a best response. Similar rules are considered for arbitrary interaction structures by Peski (2010).

[4] See Sawa and Wu for a discussion of their own related work (Sawa, 2015; Sawa and Wu, 2018b).

[5] Regarding switching rates, Norman (2009b) shows that convergence to risk dominant strategies can actually be speeded up when switching rates under a perturbed best response dynamic are decreased for small payoff differences. Wood (2014) considers robustness to several behavioral biases in the beliefs of best responding agents and shows that, while selection results remain robust, convergence rates can be faster than is the case in the absence of these biases. Newton and Angus (2015) show that coalitional behavior can greatly speed or slow convergence rates and that these effects can depend on knife-edge payoff differences.

[6] This means we avoid having to assume that n is large to guarantee that a single state is stochastically stable. Sawa and Wu (2018a) assume that n is large to deal with odd-valued n but there is ambiguity in both their limiting definition of stochastic stability for large n and the part of the proof of Theorem 3.2 that deals with large n.

	A	B
A	a_{AA}	a_{AB}
B	a_{BA}	a_{BB}

(i)

	A	B
A	$u^x(a_{AA})$	$u^x(a_{AB})$
B	$u^x(a_{BA})$	$u^x(a_{BB})$

(ii)

Fig. 1. Payoffs and utilities. $a_{AA} > a_{BA}$ and $a_{BB} > a_{AB}$. **Panel (i):** For each combination of A and B, entries give payoffs for the row player. **Panel (ii):** At state x, the row player assesses payoffs from the game in Panel (i) according to the strictly increasing utility function $u^x(\cdot)$.

$$P_\varepsilon(x, x) = 1 - P_\varepsilon\left(x, x + \frac{1}{n}\right) - P_\varepsilon\left(x, x - \frac{1}{n}\right).$$

As any state can be reached from any other state, P_ε is irreducible and has a unique invariant measure, which we denote π_ε. If $\pi_\varepsilon(x) \to 1$ as $\varepsilon \to 0$, we say that state x is uniquely *stochastically stable* (Foster and Young, 1990).

3. Utility transformations without state dependence

Consider state-independent preferences. That is, $u^x(\cdot)$ is constant in x. We write $u^x(\cdot) = u(\cdot)$. Thus we can rewrite (2.1) as

$$U^x(s) = (1 - x)\,u(a_{sA}) + x\,u(a_{sB}). \tag{3.1}$$

Note that, for $x = \frac{1}{2}$, we have that $U^x(A) > U^x(B)$ if and only if

$$u(a_{AA}) + u(a_{AB}) > u(a_{BA}) + u(a_{BB}). \tag{3.2}$$

This is the condition for strict risk dominance of strategy A under utility u. Consequently, if (3.2) holds, then $U^x(A) > U^x(B)$ for all $x \in \{0, \ldots, \frac{1}{2}\}$. Thus at least $\frac{n}{2} + 1$ deviations are required for the process to transit from $x = 0$ to $x = 1$, whereas at most $\frac{n}{2}$ deviations are required for the process to transit from $x = 1$ to $x = 0$. Consequently, $\pi_\varepsilon(0) \to 1$ as $\varepsilon \to 0$. State $x = 0$ is uniquely stochastically stable.

Conditions can be given for the underlying game of Fig. 1[i] that make A strictly risk dominant in the transformed game of Fig. 1[ii]. Firstly, if A is risk dominant and maximin in the underlying game, then A is risk dominant in the transformed game for any concave utility function.

Proposition 1. *(3.2) holds for every concave $u(\cdot)$ if and only if*

(i) $a_{AA} + a_{AB} > a_{BA} + a_{BB}$ (strict risk dominance), and
(ii) $a_{AB} \geq a_{BA}$ (maximin).

Proof.
Necessity.

If (i) does not hold, then any linear (therefore concave) u will not satisfy (3.2).

If (ii) does not hold, then $a_{BA} > a_{AB}$ and it must be that $a_{AB} < \min\{a_{AA}, a_{BA}, a_{BB}\}$. Let $u(a) = a - \min\{a_{AA}, a_{BA}, a_{BB}\}$ for $a \geq \min\{a_{AA}, a_{BA}, a_{BB}\}$, and $u(a) = -\lambda(\min\{a_{AA}, a_{BA}, a_{BB}\} - a)$ for $a < \min\{a_{AA}, a_{BA}, a_{BB}\}$. Note that $u(\cdot)$ is concave for $\lambda \geq 1$. For large enough λ, (3.2) does not hold.
Sufficiency. Let (i) and (ii) hold and $u(\cdot)$ be concave.

If $a_{AA} > a_{BB}$, then $a_{AA} > a_{BB} > a_{AB} \geq a_{BA}$, so (3.2) holds as $u(\cdot)$ is strictly increasing.

If $a_{AA} \leq a_{BB}$, then $a_{BB} > a_{AB} \geq a_{AA} > a_{BA}$ or $a_{BB} \geq a_{AA} \geq a_{AB} \geq a_{BA}$. Define u' so that $u'(a) = u(a)$ everywhere except on the interval $[\min\{a_{AA}, a_{AB}\}, \max\{a_{AA}, a_{AB}\}]$ on which $u'(\cdot)$ is given by the chord from $(a_{AA}, u(a_{AA}))$ to $(a_{AB}, u(a_{AB}))$. As the minimum of two concave functions (u and a straight line), u' is concave. Note that $u'(a) = u(a)$ for $a = a_{AA}, a_{AB}, a_{BA}, a_{BB}$. It follows that

$$\frac{1}{2}u(a_{BB}) + \frac{1}{2}u(a_{BA}) = \frac{1}{2}u'(a_{BB}) + \frac{1}{2}u'(a_{BA}) \underbrace{\leq}_{\text{by concavity}} u'\left(\frac{1}{2}a_{BB} + \frac{1}{2}a_{BA}\right)$$

$$\underbrace{<}_{\text{by (i)}} u'\left(\frac{1}{2}a_{AA} + \frac{1}{2}a_{AB}\right) = \frac{1}{2}u'(a_{AA}) + \frac{1}{2}u'(a_{AB}) = \frac{1}{2}u(a_{AA}) + \frac{1}{2}u(a_{AB}),$$

which completes the argument. □

The conditions in Proposition 1, taken together, comprise what Sawa and Wu (2018a) call 'loss dominance'. We have shown that these conditions are needed to ensure that strategy A remains risk dominant under concave (i.e. risk averse) utility transformations, even without state dependence. Furthermore, a similar argument proves an analogous result for convex (i.e. risk seeking) utility transformations.

Proposition 2. *(3.2) holds for every convex $u(\cdot)$ if and only if*

(i) $a_{AA} + a_{AB} > a_{BA} + a_{BB}$ *(strict risk dominance), and*
(ii) $a_{AA} \geq a_{BB}$ *(payoff dominance).*

The proof of Proposition 2 is relegated to the appendix as it has a similar structure to the proof of Proposition 1. Note that weak variants of Propositions 1 and 2 can be proved if we replace the strict inequalities of Condition (i) and expression (3.2) with weak inequalities.

The difference in the conditions of Propositions 1 and 2 arises from the following. Concave utility can make bad payoffs relatively worse, so to ensure that strategy A remains risk dominant under such utility, a maximin condition is required to ensure that the smallest possible payoff in the payoff matrix is not associated with A. Conversely, convex utility can make good payoffs relatively better, so to ensure that strategy A remains risk dominant under such utility, a payoff dominance condition is required to ensure that the largest possible payoff in the payoff matrix is associated with A.

4. State-dependent utility

Now consider state-dependent u^x and let (3.2) hold for all x. That is,

$$\text{For all } x, \quad u^x(a_{AA}) + u^x(a_{AB}) > u^x(a_{BA}) + u^x(a_{BB}). \tag{4.1}$$

In words, A is risk dominant in the transformed game under the utility function at any state. This would be the case, for example, if u^x were concave and the conditions of Proposition 1 held. Risk dominance of A at every state implies that $U^x(A) > U^x(B)$ for all $x \in \{0, \dots, \frac{1}{2}\}$. As before, at least $\frac{n}{2} + 1$ deviations are required for the process to transit from $x = 0$ to $x = 1$, whereas at most $\frac{n}{2}$ deviations are required for the process to transit from $x = 1$ to $x = 0$. Once again, $x = 0$ is uniquely stochastically stable.

The reason that state dependence does not change the implications of the model is that the choice probabilities in (2.2) depend only on the ordinal ranking of $U^x(A)$ and $U^x(B)$. When risk dominance is preserved by u^x, this ordinal ranking is determined to be $U^x(A) > U^x(B)$ for all $x \in \{0, \dots, \frac{1}{2}\}$ and choice probabilities at these states are exactly as before.

5. Example: prospect theory preferences

Sawa and Wu (2018a) consider the following utility function

$$u^x(a) := \begin{cases} (a - r^x)^\alpha & \text{if } a - r^x \geq 0, \\ -\lambda\,(r^x - a)^\alpha & \text{otherwise,} \end{cases} \tag{5.1}$$

where $\{r^x\}_{x \in X}$, $r^x \in \mathbb{R}$ are state-dependent reference points, $\alpha \in (0, 1]$ represents diminishing sensitivity to gains and losses, and $\lambda \geq 1$ represents the degree of loss aversion. If $\alpha = 1$, then u^x are piecewise-linear and concave. Moreover, the counterexample in the proof of the 'Necessity' part of Proposition 1 uses a utility function that satisfies (5.1) with $r^x = \min\{a_{AA}, a_{BA}, a_{BB}\}$ and $\alpha = 1$. From Proposition 1 and our discussion of state dependence, we therefore recover the main theorem of Sawa and Wu (2018a).[7]

Theorem SW. *(Theorem 3.2 of Sawa and Wu, 2018a)*
$x = 0$ is uniquely stochastically stable for all utilities given by (5.1) with $\alpha = 1$ if and only if

(i) $a_{AA} + a_{AB} > a_{BA} + a_{BB}$ *(strict risk dominance), and*
(ii) $a_{AB} \geq a_{BA}$ *(maximin).*

Loss aversion ($\lambda \geq 1$) has played no role beyond, in conjunction with constant sensitivity to gains and losses ($\alpha = 1$), ensuring that (5.1) represents risk averse preferences at every state. To see beyond doubt that it is risk aversion that relates to the loss dominance conditions, now consider the case when strategy A satisfies (i) and (ii) of Theorem SW, but does not

[7] Restricting attention to subclasses of concave utility can weaken the conditions for (3.2) to hold. For example, if we restrict attention to linear utility, then we only require condition (i) of Proposition 1. Theorem SW shows that the class of utilities given by (5.1) with $\alpha = 1$ does not lead to such a weakening. However, Sawa and Wu (2018a, Theorem 5.1) show that weaker conditions apply under the additional restriction that r^x equals the average payoff over all players.

satisfy payoff dominance. That is, we have $a_{BB} > a_{AA}$. Consider preferences given by (5.1) with $r^x \equiv a_{BB}$. It is clear that, for small enough α (that is, when utility is sufficiently S-shaped), inequality (3.2) is reversed. The converse of the argument that follows (3.2) then implies that $x = 1$ is uniquely stochastically stable. Importantly, by setting $r^x \equiv a_{BB}$ we have been able to use the convex part of the utility curve, ensuring that Proposition 1 does not apply.[8]

Acknowledgments

The authors gratefully acknowledge the comments of the editor (Françoise Forges), the associate editor, two anonymous referees, Ennio Bilancini, Leonardo Boncinelli, Ryoji Sawa and Jiabin Wu. Nax's work was supported by the European Commission through the ERC Advanced Investigator Grant 'Momentum' (Grant 324247). Newton's work was supported by a KAKENHI Grant-in-Aid for Research Activity Start-Up funded by the Japan Society for the Promotion of Science (Grant 18H05680); and the Research Fund for Young Scientists of Kyoto University (Grant 38099300).

Appendix A. Remaining proofs

Proof of Proposition 2.
Necessity.

If (i) does not hold, then any linear (therefore convex) u will not satisfy (3.2).

If (ii) does not hold, then $a_{BB} > a_{AA}$ and it must be that $a_{BB} > \max\{a_{AB}, a_{BA}, a_{AA}\}$. Let $u(a) = a - \max\{a_{AB}, a_{BA}, a_{AA}\}$ for $a \leq \max\{a_{AB}, a_{BA}, a_{AA}\}$, and $u(a) = \lambda\,(a - \max\{a_{AB}, a_{BA}, a_{AA}\})$ for $a > \max\{a_{AB}, a_{BA}, a_{AA}\}$. Note that $u(\cdot)$ is convex for $\lambda \geq 1$. For large enough λ, (3.2) does not hold.

Sufficiency. Let (i) and (ii) hold and $u(\cdot)$ be convex.

If $a_{AB} > a_{BA}$, then $a_{AA} \geq a_{BB} > a_{AB} > a_{BA}$, so (3.2) holds as $u(\cdot)$ is strictly increasing.

If $a_{AB} \leq a_{BA}$, then $a_{AA} > a_{BA} \geq a_{BB} > a_{AB}$ or $a_{AA} \geq a_{BB} \geq a_{BA} \geq a_{AB}$. Define u' so that $u'(a) = u(a)$ everywhere except on the interval $[\min\{a_{BA}, a_{BB}\}, \max\{a_{BA}, a_{BB}\}]$ on which $u'(\cdot)$ is given by the chord from $(a_{BA}, u(a_{BA}))$ to $(a_{BB}, u(a_{BB}))$. As the maximum of two convex functions (u and a straight line), u' is convex. Note that $u'(a) = u(a)$ for $a = a_{AA}, a_{AB}, a_{BA}, a_{BB}$. It follows that

$$\frac{1}{2}u(a_{BB}) + \frac{1}{2}u(a_{BA}) = \frac{1}{2}u'(a_{BB}) + \frac{1}{2}u'(a_{BA}) = u'\left(\frac{1}{2}a_{BB} + \frac{1}{2}a_{BA}\right)$$
$$\underbrace{<}_{\text{by (i)}} u'\left(\frac{1}{2}a_{AA} + \frac{1}{2}a_{AB}\right) \underbrace{\leq}_{\text{by convexity}} \frac{1}{2}u'(a_{AA}) + \frac{1}{2}u'(a_{AB}) = \frac{1}{2}u(a_{AA}) + \frac{1}{2}u(a_{AB}),$$

which completes the argument. □

References

Bergin, J., Lipman, B.L., 1996. Evolution with state-dependent mutations. Econometrica 64, 943–956. http://ideas.repec.org/a/ecm/emetrp/v64y1996i4p943-56.html.
Bilancini, E., Boncinelli, L., 2019. The evolution of conventions under condition-dependent mistakes. Econ. Theory. https://doi.org/10.1007/s00199-019-01174-y, forthcoming.
Blume, L.E., 2003. How noise matters. Games Econ. Behav. 44, 251–271.
Foster, D., Young, H.P., 1990. Stochastic evolutionary game dynamics. Theor. Popul. Biol. 38, 219–232.
Hwang, S.H., Lim, W., Neary, P., Newton, J., 2018. Conventional contracts, intentional behavior and logit choice: equality without symmetry. Games Econ. Behav. 110, 273–294. https://doi.org/10.1016/j.geb.2018.05.002.
Kahneman, D., Tversky, A., 1984. Choices, values, and frames. Am. Psychol. 39, 341–350.
Kandori, M., Mailath, G.J., Rob, R., 1993. Learning, mutation, and long run equilibria in games. Econometrica 61, 29–56. http://ideas.repec.org/a/ecm/emetrp/v61y1993i1p29-56.html.
Lim, W., Neary, P.R., 2016. An experimental investigation of stochastic adjustment dynamics. Games Econ. Behav. 100, 208–219.
Maruta, T., 2002. Binary games with state dependent stochastic choice. J. Econ. Theory 103, 351–376.
Mäs, M., Nax, H.H., 2016. A behavioral study of in coordination games. J. Econ. Theory 162, 195–208. https://doi.org/10.1016/j.jet.2015.12.010.
Newton, J., 2018. Evolutionary game theory: a renaissance. Games 9, 31. https://doi.org/10.3390/g9020031. http://www.mdpi.com/2073-4336/9/2/31.
Newton, J., Angus, S.D., 2015. Coalitions, tipping points and the speed of evolution. J. Econ. Theory 157, 172–187. https://doi.org/10.1016/j.jet.2015.01.003.
Norman, T., 2009a. Skew-symmetry under simultaneous revisions. Int. Game Theory Rev. 11, 471–478.
Norman, T.W., 2009b. Rapid evolution under inertia. Games Econ. Behav. 66, 865–879.
Peski, M., 2010. Generalized risk-dominance and asymmetric dynamics. J. Econ. Theory 145, 216–248. https://doi.org/10.1016/j.jet.2009.05.007.
Sawa, R., 2015. Prospect theory Nash bargaining solution and its stochastic stability. Mimeo.
Sawa, R., Wu, J., 2018a. Prospect dynamics and loss dominance. Games Econ. Behav.. https://doi.org/10.1016/j.geb.2018.07.006 (online first).

[8] As (5.1) is continuous in α and conditions (i) and (ii) imply that the strict inequality (3.2) holds, we have that (3.2) continues to hold in some neighborhood of $\alpha = 1$ and, therefore, so do the implications of Theorem SW. This robustness result is Theorem 6.1 of Sawa and Wu (2018a), although their proof differs from the argument given here.

Sawa, R., Wu, J., 2018b. Reference-dependent preferences, super-dominance and stochastic stability. J. Math. Econ. 78, 96–104. https://doi.org/10.1016/j.jmateco.2018.08.002.

Weinstein, J., 2016. The effect of changes in risk attitude on strategic behavior. Econometrica 84, 1881–1902.

Wood, D.H., 2014. The evolution of behavior in biased populations. Mimeo.

Young, H.P., 1993. The evolution of conventions. Econometrica 61, 57–84. http://ideas.repec.org/a/ecm/emetrp/v61y1993i1p57-84.html.